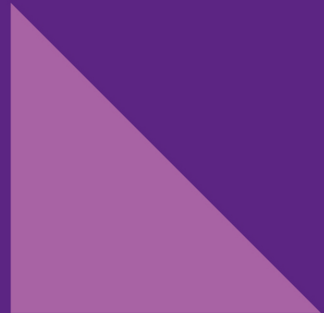


Sterile Neutrino Dark Matter

Alexander Merle



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To Werner Maneschg—who predicted that years ago

Contents

Preface	ix
Acknowledgments	x
About the author	xi
1 Introduction: Dark Matter—what we do and do not know	1-1
1.1 Observational evidence for Dark Matter	1-1
1.2 Explanations for Dark Matter	1-2
1.3 Sterile neutrinos as Dark Matter	1-4
References	1-5
2 Sterile neutrinos—almost part of the Standard Model	2-1
2.1 Fermion masses in the Standard Model	2-1
2.2 Why neutrino masses seem peculiar	2-3
2.3 Explaining sterile neutrino masses and mixings	2-7
References	2-10
3 Dark Matter—fossils from the early Universe	3-1
3.1 General thoughts on Dark Matter production	3-1
3.2 Thermal freeze-out	3-4
3.3 Non-thermal distribution functions	3-8
3.4 Production mechanisms for keV sterile neutrinos	3-9
References	3-11
4 A very big small effect—production by active-sterile mixing	4-1
4.1 The freeze-in mechanism	4-2
4.2 Non-resonant production: Dodelson–Widrow mechanism	4-3
4.2.1 Producing all sterile neutrinos by DW	4-4
4.2.2 The DW mechanism as secondary process	4-7
4.3 Resonant production: Shi–Fuller mechanism	4-8
4.3.1 How to hit a narrow peak	4-8
4.3.2 A simple but complete setting: the ν MSM	4-10
References	4-12

5	Resurrection from the downfall—production by particle decays	5-1
5.1	Two-step production of Dark Matter	5-1
5.2	Scalar freezing in	5-3
5.3	Scalar freezing out	5-6
	5.3.1 Early decay	5-7
	5.3.2 Late decay and intermediate regime	5-8
5.4	The Dodelson–Widrow modification	5-10
	References	5-10
6	The emergence of order—cosmic structure formation	6-1
6.1	The Tremaine–Gunn bound	6-2
6.2	The free-streaming horizon	6-3
6.3	The evolution equations for cosmic structure formation	6-5
6.4	The matter power spectrum	6-8
6.5	Bounds from and implications for structure formation	6-10
	References	6-14
7	Consult the stars for an answer—astrophysical signals	7-1
7.1	The radiative decay of the sterile neutrino	7-2
7.2	Pulsar kicks	7-4
7.3	Bounds from supernovae	7-6
7.4	Putting all astrophysical constraints together	7-7
	References	7-10
8	The needle in the dark haystack—experimental attempts	8-1
8.1	Single beta decay	8-2
8.2	Electron capture decays	8-3
8.3	Sterile neutrino capture on stable nuclei	8-5
8.4	Drawing conclusions from active-neutrino experiments and getting a global picture	8-6
	References	8-9
9	What to take home—conclusions and outlook	9-1

Preface

Prerequisites

In order to gain something from this book, a potential reader should be somewhat familiar with the Standard Model of elementary particle physics. In addition, they should have some basic knowledge of cosmology. While we will not attempt to do very complicated computations, it will nevertheless often be very useful to be accustomed to certain notions used in both fields. For example, an ideal reader would know what ‘comoving momentum’ means, they should roughly understand how to read off an interaction from a Lagrangian, and certainly some background knowledge on WIMP Dark Matter would also be useful. Similarly, some very generic conventions used in nearly any text on the Standard Model, such as employing natural units (i.e. $\hbar = c = 1$), will be used without further explanation. Finally, although not strictly necessary, some knowledge of neutrino physics can be of advantage. Nevertheless, I have tried to keep the text as basic as possible, keeping in mind that it is supposed to be a concise first introduction to a specialised topic.

Books that could be useful to have read are, e.g. *An Introduction to Quantum Field Theory* by Peskin and Schroeder for elementary particle physics or *The Early Universe* by Kolb and Turner for cosmology. Nevertheless, I will in any chapter point out useful references which treat the topics discussed in the respective chapter in greater detail. Given that there exist no textbooks for several of the aspects discussed, some of these references will be actual research papers. However, at least for some topics, excellent review articles are already available. While it is clear that, within the pre-determined format, no fully comprehensive discussion can be given, this text should nevertheless serve as an overview and hopefully an appetiser for the reader to explore the topic themselves. As for further reading, probably the richest document available, collecting information from all sides involved, is Adhikari *et al* 2017 A White Paper on keV sterile neutrino Dark Matter *J. Cosmol. Astropart. Phys.* [JCAP01\(2017\)025](#).

Acknowledgments

First and foremost, my gratitude goes towards Nicki Dennis from Morgan & Claypool Publishers, for asking me to write a book on such a rather specialised subject. While this thought had not crossed my mind prior to that, the whole concept of writing a concise and pedagogical introduction to the field strongly appealed to me. Special thanks goes also to Karen Donnison, who took care of the permissions for some of the figures, which saved me a lot of trouble. Naturally, in the course of writing this book, I have drawn a lot from work I did with many great collaborators on keV sterile neutrinos and on related topics, among them Pasquale Di Bari, Marco Drewes, Steve King, Johannes König, Thierry Lasserre, Manfred Lindner, Nicola Menci, Susanne Mertens, Alessandro Mirizzi, Stefano Morisi, Ninetta Saviano, Daniel Schmidt, Matteo Viel, and Walter Winter. An even greater thank you, for several wonderful works done together on keV neutrinos and for being the most perfect collaborators I ever had the pleasure to work with, is deserved by Viviana Niro, Aurel Schneider, and Max Totzauer—and both Aurel and Max I also cannot thank enough for countless useful comments on the manuscript for this book. Finally, I am grateful to Kev Abazajian, Loredana Gastaldo, Thomas Tram, and Teja Venumadhav for sharing information on their work.

However, the greatest thank you of all, and all my love, is reserved for my family: my wife Martina, my son Atreyu, and my daughter Melissa. While I truly cannot claim that my kids have done very much to help me completing this book, they, together with my wife, are certainly responsible for the most joyful moments of my life.

THANK YOU!!!

About the author

Alexander Merle



Alexander Merle was born in Arad, Romania, close to the famous region of Transylvania. After his emmigration to Germany at the tender age of two months, he grew up in the middle of Bavaria before entering the academic world. He is old enough to still hold the by now discontinued degree of a ‘Diplom-Physiker’ (Dipl.-Phys. Univ.) from Munich University of Technology in 2006. He obtained his PhD from Heidelberg University in 2009. He left Germany to be a postdoc in Stockholm, Sweden, before he progressed by securing a Marie Curie Fellowship with the University of Southampton, United Kingdom. He subsequently returned to Germany where he took on a Senior Postdoc position at the Max Planck Institute for Physics (Werner Heisenberg Institute) in Munich, Germany, where he is supervising a small group of PhD and Masters students.

Dr Merle’s main research fields are theoretical elementary particle physics and cosmology, with a particular focus on neutrinos, Dark Matter, and their inter-connections. He has written more than 60 papers on various topics, some even reaching out to neighboring fields such as mathematics or condensed matter physics, and some being cross-field collaborations with colleagues from, e.g. experimental physics or astronomy. He is an active contributor to the field of keV sterile neutrino Dark Matter, having discovered the production mechanism (FIMP scalar decay) that is currently in best agreement with data, having written a topcite review on model building aspects of keV sterile neutrinos, and having made several proposals on how to corner these particles with contemporary experiments. His reputation in the field is reflected by him being one of four main editors, section editor, and corresponding author of Adhikari *et al* A White paper on keV sterile neutrino Dark Matter *J. Cosmol. Astropart. Phys.* [JCAP01\(2017\)025](#), which is the most comprehensive document that exists on the topic.

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Chapter 1

Introduction: Dark Matter—what we do and do not know

Modern physics has developed in a way that sounds incredible at times: indeed, many phenomena and observations which researchers discuss on a daily basis may appear truly unusual to somebody learning about them for the first time. For example, the curious antimatter, being a substance that destroys ordinary matter and creates radiation from it, does indeed sound like science fiction at first sight. Yet, not only have we established its existence, we even have come up with applications in fields as down-to-Earth as medicine, where positron (i.e. antimatter) emission tomography is used to detect cancerous cells in the human body.

In contemporary cosmology, we are dealing with several such mysteriously sounding substances with peculiar properties. Probably the biggest mystery we are currently facing is the question of what is behind the Dark Matter in the Universe. Dark Matter is a substance that we cannot see or feel, because we are naturally bad at sensing gravity of small bodies. Nevertheless, we know that Dark Matter is in fact much more generic in the Universe than ‘ordinary’ matter, i.e. baryons, atoms, electrons—basically everything we and other objects in the Universe such as planets, stars, or galaxies consist of. Indeed, about 80% of the matter content of our Universe is ‘dark’, and we have no idea what could be behind this unknown substance.

1.1 Observational evidence for Dark Matter

How come we are so convinced of the existence of Dark Matter? Due to a multitude of reasons. In fact, we only observe Dark Matter through its gravitational effects—but this has multiple consequences. First, the motions of galaxies and whole galaxy clusters are influenced by the presence of Dark Matter. A famous example is the rotational velocity of spiral galaxies, as a function of the distance from their centre, from which one can infer their mass-to-light ratio. Without the existence of Dark Matter, with only visible matter being present, this rotational velocity should steeply

increase when radially departing from the centre, and reach a maximum value before falling off. However, the data tell us a different story: instead of declining, the rotational velocity remains nearly constant after reaching its maximum, down to the outer arms of the galaxy. This behaviour can be explained by the presence of an invisible form of matter: *Dark Matter*. Further evidence arises from other observations, such as the motions of objects of different sizes, spanning from stars (studied by Jan Oort) to whole galaxy clusters (studied by Fritz Zwicky).

Yet, this alone would probably convince no real scientist: for example, we could think of a modification of the law of gravity on such large scales, which has indeed been proposed as an alternative explanation for the observed behaviour. However, in 2006, an observation of two colliding clusters of galaxies was made public, the so-called Bullet Cluster. This provided the ultimate game changer: the reconstruction of the Dark Matter distribution within the clusters, as achieved by gravitational lensing, clearly showed that Dark Matter is strongly favoured over any alternative explanations for the observed clash of galaxies, to the point that practically no scientist had serious doubts about its existence anymore.

The most precise determination of the amount of Dark Matter in the Universe is derived from the peaks in the spatial correlations of anisotropies in the *cosmic microwave background*, electromagnetic radiation from which was produced when the Universe was about 380 000 years old and which has filled it ever since. The first very precise determinations of the amount of Dark Matter have correspondingly been put forward by two microwave satellites a few decades ago, the Cosmic Background Explorer (COBE) and the Wilkinson Microwave Anisotropy Probe (WMAP). Nowadays, the 2015 data set from the Planck satellite [1], along with complementary observations such as the expansion rate of the Universe, constrain the Dark Matter abundance to be $\Omega_{\text{DM}}h^2 = 0.1188 \pm 0.0010$, a value determined with remarkable precision. This means that the energy density of the Dark Matter today, compared to the total energy density of the Universe, is given by $\Omega_{\text{DM}} = 25.89\%$, where $h = 0.673$ is the reduced Hubble constant, i.e. H_0 in units of $100 \text{ (km s}^{-1}\text{) Mpc}^{-1}$. We can say that about 1/4 of our current Universe consists of Dark Matter, with the rest made up by ordinary matter (4.86%) and dark energy (69.11%). Only a small fraction ($<0.5\%$) is present in the form of radiation, i.e. photons and neutrinos.

1.2 Explanations for Dark Matter

Given the existence of Dark Matter, the natural follow-up question is what it consists of. Sticking to what we know, the most generic attempt is to explain it by something that we have already detected. Good candidates for massive objects that are not bright, in the sense that they do not produce radiation detectable by optical telescopes, would for example be big gas planets similar to Jupiter. Such objects, called MACHOs (MASSive Compact Halo Objects), are just a little too small for nuclear fusion to be triggered in their centres, so they allow ‘hiding’ a lot of mass inside them without producing visible light. However, such objects would not be fully invisible, since they block and influence visible light after all, so that they would

leave imprints on the light of stars whose line of sight the MACHOs may cross. On the other hand, while MACHOs can still make up a fraction (but not all) of the Dark Matter present in galaxies, it seems very unlikely that they could be behind the effects of Dark Matter observed in the cosmic microwave background, since at that time the Universe was still too hot for baryons to form big bound structures such as MACHOs [3]. Similar arguments can be given against other attempts for an explanation of Dark Matter by astrophysical objects. Thus, other alternatives such as relatively light black holes, produced by some unknown mechanism in the early Universe, also seem to be unlikely [5].

There is a very natural possibility to nevertheless explain Dark Matter, though, in a way that does not seem challenged by any observation: it could be an elementary particle [2]. We have found many different elementary particles in Nature, and if any of them (or one that has not been found yet) could make up the Dark Matter, we may even be able to derive testable predictions and trigger experimental searches for this particular particle. It turns out that, in order to act as Dark Matter, a particle needs to tick several boxes. First, it needs to be electrically neutral (in fact, in order to be sufficiently abundant, at most weakly interacting), as otherwise it would interact too strongly with light and thus not be dark at all. Second, it should be massive, because we have observed Dark Matter by its gravitational effects. Third, and this is a very non-trivial requirement, the Dark Matter candidate particle has to have been produced in the early Universe in the right amount to make up a sizeable fraction (ideally all) of the Dark Matter observed, and the velocity or momentum distribution $f(p, T)$ of the particles must not feature too many particles with very large momenta, as this would have prevented galaxies from forming. Finally, Dark Matter needs to be stable enough to still be around today, or at least not have disappeared in large amounts.

In the Standard Model of elementary particle physics, no suitable particle exists. Out of all electrically neutral and massive particles, only neutrinos may possibly contribute to the Dark Matter, but their small mass and large velocities prevent them from being more than a small fraction of it. Fortunately, this picture changes once we extend the Standard Model by new physics. Historically, a breakthrough arose from an idea proposed by Lee and Weinberg in 1977 [6]: assuming that heavy stable neutrinos exist, they showed that such particles could easily be produced in the early Universe. At high temperatures T , when the Universe was very dense and small, such particles would be in ‘thermal equilibrium’ with all Standard Model particles; i.e. the Dark Matter particles would annihilate with a large rate into known matter, so large it even supersedes the expansion rate of the Universe. As the latter cools down, however, all species are diluted and, at some point, the particle density is so low that the Dark Matter particles no longer find any annihilation partners. Thus, given that they are stable, they have remained present in the Universe until today. This mechanism is called *thermal freeze-out*, and it can be used to produce a sizable amount of Dark Matter particles [4]. In order to meet the correct abundance, these particles typically need masses of a few 100 GeV. In modern language, particles with such properties are called *WIMPs* (Weakly Interacting Massive Particles). The fact that interaction strengths similar to that of

the weak interaction are, by Dark Matter production, connected to the electroweak scale (where we anyway expect new physics to be present) is often dubbed the *WIMP miracle*.

While a WIMP may be quite a general category of particles, it is remarkable that several theories beyond the Standard Model do predict WIMP-candidates: for example, models based on supersymmetry generically feature an electrically neutral spin-1/2 Majorana fermion called the neutralino, which is in many cases stable and interacts roughly with weak-scale cross sections. Another option is theories based on extra spatial dimensions, which also predict particles with the characteristics of WIMPs. Note that, due to the details of the production process and the requirement of not producing too much Dark Matter, WIMPs typically act as what is called *cold* Dark Matter: they have non-relativistic speeds from the moment they are produced. Phenomenologically, WIMPs are interesting due to their potential to be discovered directly (by their interactions with nuclei), indirectly (by detecting their annihilation products), or even by producing them at particle colliders. A WIMP is a very good Dark Matter candidate indeed, with the only serious problem that, up to now, experiments have considerably narrowed down the viable parameter space for WIMPs *without*, however, a clear discovery (at most hints inconsistent with each other have been seen).

However, particle physicists do not run out of ideas easily. Many alternatives, non-WIMP Dark Matter candidates have been proposed. Some of these candidates have the practical problem that they are hardly testable—in the sense that, while they may exist and be the Dark Matter, they would be so feebly interacting that we may have no chance to ever detect them. While such an option is of course not logically forbidden, the only categories of particles we could possibly test are those with observable signatures. Thus, we would ideally like to link Dark Matter particles to something that we already know. In the case of WIMPs, that something is the weak interaction itself. There are, however, at least three other popular candidate particles that are very unlike WIMPs, but nevertheless connected to known sectors of physics: the *axion* is a very light and very feebly interacting spin-0 scalar that may be connected to the so-called strong CP problem; the spin-3/2 *gravitino* is the so far undiscovered supersymmetric partner of the graviton, the quantum field of gravitation; and the spin-1/2 *sterile neutrino* is connected to the existence of right-handed neutrino fields, which are singlets under the Standard Model gauge group and whose existence is ultimately linked to neutrinos being massive.

1.3 Sterile neutrinos as Dark Matter

A sterile neutrino is an up-to-now hypothetical fermion that is a total singlet with respect to the Standard Model, i.e. it is perfectly neutral. Mathematically, it is introduced by adding so-called ‘right-handed’ neutrino fields to the Standard Model, which are singlets $(\mathbf{1}, \mathbf{1}, 0)$ under the full $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. Such a field can have a Majorana mass term, which implies that it is identical to its anti-particle. Furthermore, the mass of a sterile neutrino is not fixed by any principal requirement from the particle physics side. In particular, it is not

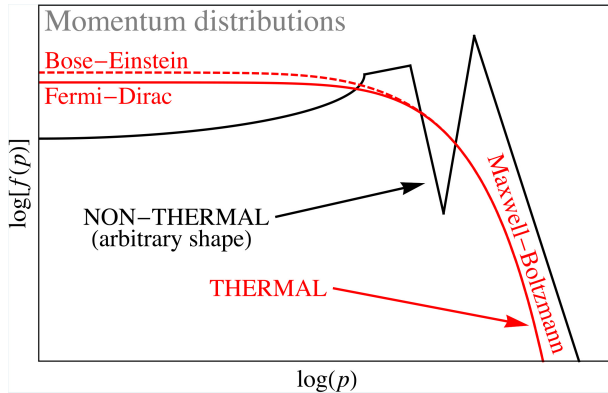


Figure 1.1. Cartoonish comparison of thermal versus non-thermal momentum distribution functions. Thermal distributions are practically featureless, and hence well described by their average momentum (p). Non-thermal distributions can have very distinct features such as spikes, kinks, or more than one dominant momentum scale.

tied to the electroweak scale in any way. However, a sterile neutrino mass eigenstate is not 100% sterile, because it mixes with ordinary (active) neutrinos. This admixture is what intimately connects sterile neutrino fields to the active neutrino sector and, as we will see, it is responsible for a lot of the important properties of sterile neutrinos.

It is this sterile neutrino as a serious candidate for Dark Matter that we will concentrate on in this book. As we will see, a sterile neutrino is a Dark Matter candidate that is somewhat peculiar, with properties very different from those of WIMPs. These new properties have a variety of consequences, which on the one hand make sterile neutrinos harder to observe, but which also connect them to other fields and ultimately yield testable predictions beyond direct detection. Also, from a cosmological point of view, sterile neutrinos do have new implications, starting from their production mechanisms generating non-thermal momentum distributions (see figure 1.1) to their non-standard predictions for the formation of structures in the Universe. We will also see how the connection of Dark Matter sterile neutrinos to active neutrinos could yield a potentially observable experimental signature of these particles, and where else they could have an influence. Hopefully—after having worked through this book—you, dearest reader, will appreciate why sterile neutrinos are very good Dark Matter candidates and what we can learn from them about one of the biggest questions of our Universe.

Let us collaborate and explore the world of sterile neutrino Dark Matter together.

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Chapter 2

Sterile neutrinos—almost part of the Standard Model

Why are researchers talking about sterile neutrinos at all? After all, while there are (albeit inconclusive!) experimental hints for sterile neutrinos with eV-scale masses [1], and possibly also for keV scale sterile neutrinos—to be discussed in chapter 7—we are far from a concrete detection of anything like a sterile neutrino. Nevertheless, from a particle physics theory perspective, sterile neutrinos are in fact quite likely to exist. Let us see why this is the case.

2.1 Fermion masses in the Standard Model

To understand active or sterile neutrino masses, we first have to understand how fermion masses are generally obtained in the Standard Model (SM) of elementary particle physics. The SM is probably the best model we have in all of science, given that its predictions are so accurate that one particular observable—the anomalous magnetic moment of the electron—can even be forecast and is experimentally verified to an amazing accuracy of ten digits. Thus, in fact, our understanding of elementary particle physics is more precise than our understanding of how planes fly! Indeed, all elementary particles we have found so far are part of the SM, even though some have only been found in the last couple of decades. While a full review of the SM would take up the space of a whole book, there is a bulk of excellent literature available. To single out two very pedagogical texts, we suggest the book [10] as a review of the full SM, and the review [5] for the Higgs mechanism in particular.

Table 2.1. Fermions in the Standard Model, along with their masses. Uncertainties are only indicated in cases where they amount to a few per cent at least.

	First generation	Second generation	Third generation
Quarks	u ($m_u = 2.3^{+0.7}_{-0.5}$ MeV)	c ($m_c = 1.275$ GeV)	t ($m_t = 173.21$ GeV)
	d ($m_d = 4.8^{+0.5}_{-0.3}$ MeV)	s ($m_s = 95 \pm 5$ MeV)	b ($m_b = 4.66$ GeV)
Leptons	ν_1^{NO} ($m_1 < 0.071$ eV)	ν_2^{NO} ($m_2 < 0.072$ eV)	ν_3^{NO} ($m_3 < 0.087$ eV)
	ν_1^{IO} ($m_1 < 0.082$ eV)	ν_2^{IO} ($m_2 < 0.083$ eV)	ν_3^{IO} ($m_3 < 0.065$ eV)
	e ($m_e = 0.510998928$ MeV)	μ ($m_\mu = 105.6583715$ MeV)	τ ($m_\tau = 1776.86$ MeV)

Let us first look at the values of the fermion masses in the SM, which are displayed in table 2.1¹. Glancing at these numbers, one can see that most of the fermions have masses at the MeV to GeV scales. Neglecting neutrinos, not even the top quark t seems to be an outlier, although it is nearly 40 times heavier than the bottom quark b . However, once we include neutrinos, although their masses are not known exactly, it is evident that they need to be lighter by more than *six* orders of magnitude than even the electron e ! In the eyes of a particle physics theorist, such a discrepancy cries out for an explanation—and, indeed, it seems unlikely that the origin of neutrino masses is the same as that for the other fermions, as we expect new physics to be involved in their generation.

But how are the masses of the other fermions generated? In order to write down a fermion mass term, we need to combine *left-handed* and *right-handed* fields. For example, an electron field e can appear in two versions, namely with left-handed, $e_L \equiv P_L e = \frac{1}{2}(1 - \gamma_5)e$, or right-handed, $e_R \equiv P_R e = \frac{1}{2}(1 + \gamma_5)e$, chirality. In a physical mass term, only combinations of left- and right-handed fields survive:

$$\mathcal{L}_{e\text{-mass}} = -m_e \bar{e}e = -m_e \bar{e}_L e_R + h.c., \quad (2.1)$$

due to $P_{L,R}P_{R,L} = 0$. However, in the SM, this is not even possible: under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, the left- and right-handed electrons have different quantum numbers, ‘ $e_L \sim (\mathbf{1}, \mathbf{2}, -1/2)$ ’ and $e_R \sim (\mathbf{1}, \mathbf{1}, -1)$. To be precise, the left-handed electron is not even an actual field in the SM (hence the inverted commas), but instead it is part of the lepton doublet L which also contains the left-handed neutrino field. In any case, it is impossible to build a gauge-invariant, i.e. total singlet, term out of the left- and right-handed electron fields. But exactly that would have been the requirement to include a term like the one in equation (2.1) in a physical Lagrangian.

¹Data are taken from the Particle Data Group [9], except for neutrinos, where we quote the limits from Planck [2], derived in combination with the global best-fit values of the neutrino mass square differences from `nu-fit.org` (v3.0) [6]; the latter are different for normal (NO: $m_1 < m_2 < m_3$) and inverted (IO: $m_3 < m_1 < m_2$) neutrino mass ordering, where $i = 1, 2, 3$ labels the *mass eigenstates* ν_i which are different from the *flavour eigenstates* ν_α , where $\alpha = e, \mu, \tau$.