Reliability-Based Mechanical Design, Volume 2
Component under Cyclic Load and Dimension Design with Required Reliability

Xiaobin Le, Wentworth Institute of Technology

A component will not be reliable unless it is designed with required reliability. Reliability-Based Mechanical Design uses the reliability to link all design parameters of a component together to form a limit state function for mechanical design. This design methodology uses the reliability to replace the factor of safety as a measure of the safe status of a component. The goal of this methodology is to design a mechanical component with required reliability and at the same time, quantitatively indicates the failure percentage of the component. Reliability-Based Mechanical Design consists of two separate books: Volume 1: Component under Static Load, and Volume 2: Component under Cyclic Load and Dimension Design with Required Reliability.

This book begins with a systematic description of a cyclic load. Then, the books use two probabilistic fatigue theories to establish the limit state function of a component under cyclic load, and further to present how to calculate the reliability of a component under a cyclic loading spectrum. Finally, the book presents how to conduct dimension design of typical components such as bar, pin, shaft, beam under static load, or cyclic loading spectrum with required reliability. Now, the designed component will be reliable because it has been designed with the required reliability.

The book presents many examples for each topic and provides a wide selection of exercise problems at the end of each chapter. This book is written as a textbook for senior mechanical engineering students after they study the course Design of Machine Elements or a similar course. This book is also a good reference for design engineers and presents design methods in such sufficient detail that those methods are readily used in the design.

ABOUT SYNTHESIS
This volume is a printed version of a work that appears in the Synthesis Digital Library of Engineering and Computer Science. Synthesis lectures provide concise original presentations of important research and development topics, published quickly in digital and print formats. For more information, visit our website: http://store.morganclaypool.com
Synthesis Lectures on Mechanical Engineering series publishes 60–150 page publications pertaining to this diverse discipline of mechanical engineering. The series presents Lectures written for an audience of researchers, industry engineers, undergraduate and graduate students.

Additional Synthesis series will be developed covering key areas within mechanical engineering.

<table>
<thead>
<tr>
<th>Title</th>
<th>Author</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability-Based Mechanical Design, Volume 2: Component under Cyclic Load and Dimension Design with Required Reliability</td>
<td>Xiaobin Le</td>
<td>2019</td>
</tr>
<tr>
<td>Reliability-Based Mechanical Design, Volume 1: Component under Static Load</td>
<td>Xiaobin Le</td>
<td>2019</td>
</tr>
<tr>
<td>Natural Corrosion Inhibitors</td>
<td>Shima Ghanavati Nasab, Mehdī Bavaheran Yāzd, Abolfazl Semnani, Homa Kakhksh, Navid Rabiee, Mohammad Rabiee, Mojtaba Bagherzadeh</td>
<td>2019</td>
</tr>
<tr>
<td>Fractional Calculus with its Applications in Engineering and Technology</td>
<td>Yi Yang and Haiyan Henry Zhang</td>
<td>2019</td>
</tr>
<tr>
<td>Essential Engineering Thermodynamics: A Student's Guide</td>
<td>Yumin Zhang</td>
<td>2018</td>
</tr>
<tr>
<td>Title</td>
<td>Author</td>
<td>Year</td>
</tr>
<tr>
<td>---------------------------------------------------------</td>
<td>-----------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Engineering Dynamics</td>
<td>Cho W.S. To</td>
<td>2018</td>
</tr>
<tr>
<td>Solving Practical Engineering Problems in Engineering Mechanics: Dynamics</td>
<td>Sayavur I. Bakhtiyarov</td>
<td>2018</td>
</tr>
<tr>
<td>Solving Practical Engineering Mechanics Problems: Kinematics</td>
<td>Sayavur I. Bakhtiyarov</td>
<td>2018</td>
</tr>
<tr>
<td>C Programming and Numerical Analysis: An Introduction</td>
<td>Seiichi Nomura</td>
<td>2018</td>
</tr>
<tr>
<td>Mathematical Magnetohydrodynamics</td>
<td>Nikolas Xiros</td>
<td>2018</td>
</tr>
<tr>
<td>Design Engineering Journey</td>
<td>Ramana M. Pidaparti</td>
<td>2018</td>
</tr>
<tr>
<td>Introduction to Kinematics and Dynamics of Machinery</td>
<td>Cho W. S. To</td>
<td>2017</td>
</tr>
<tr>
<td>Microcontroller Education: Do it Yourself, Reinvent the Wheel, Code to Learn</td>
<td>Dimosthenis E. Bolanakis</td>
<td>2017</td>
</tr>
<tr>
<td>Unmanned Aircraft Design: A Review of Fundamentals</td>
<td>Mohammad Sadraey</td>
<td>2017</td>
</tr>
<tr>
<td>Introduction to Refrigeration and Air Conditioning Systems: Theory and Applications</td>
<td>Allan Kirkpatrick</td>
<td>2017</td>
</tr>
</tbody>
</table>
Resistance Spot Welding: Fundamentals and Applications for the Automotive Industry
Menachem Kimchi and David H. Phillips
2017

MEMS Barometers Toward Vertical Position Detection: Background Theory, System Prototyping, and Measurement Analysis
Dimosthenis E. Bolanakis
2017

Engineering Finite Element Analysis
Ramana M. Pidaparti
2017
Reliability-Based Mechanical Design
Volume 2
Component under Cyclic Load and Dimension Design with Required Reliability

Xiaobin Le
Wentworth Institute of Technology

SYNTHESIS LECTURES ON MECHANICAL ENGINEERING #21
ABSTRACT
A component will not be reliable unless it is designed with required reliability. *Reliability-Based Mechanical Design* uses the reliability to link all design parameters of a component together to form a limit state function for mechanical design. This design methodology uses the reliability to replace the factor of safety as a measure of the safe status of a component. The goal of this methodology is to design a mechanical component with required reliability and at the same time, quantitatively indicates the failure percentage of the component. *Reliability-Based Mechanical Design* consists of two separate books: *Volume 1: Component under Static Load*, and *Volume 2: Component under Cyclic Load and Dimension Design with Required Reliability*.

This book is *Reliability-Based Mechanical Design, Volume 2: Component under Cyclic Load and Dimension Design with Required Reliability*. It begins with a systematic description of a cyclic load. Then, the books use two probabilistic fatigue theories to establish the limit state function of a component under cyclic load, and further to present how to calculate the reliability of a component under a cyclic loading spectrum. Finally, the book presents how to conduct dimension design of typical components such as bar, pin, shaft, beam under static load, or cyclic loading spectrum with required reliability. Now, the designed component will be reliable because it has been designed with the required reliability.

The book presents many examples for each topic and provides a wide selection of exercise problems at the end of each chapter. This book is written as a textbook for senior mechanical engineering students after they study the course Design of Machine Elements or a similar course. This book is also a good reference for design engineers and presents design methods in such sufficient detail that those methods are readily used in the design.

KEYWORDS
reliability, reliability-based design, mechanical component, mechanical design, computational method, numerical simulation, static load, cyclic load, fatigue damage, limit state function, failure, safety, probability, the P-S-N curve approach, the K-D probabilistic fatigue damage model
To my lovely wife, Suyan Zou,
and to my wonderful sons, Zelong and Linglong
# Contents

**Preface** ................................................................. xv

1  **Introduction and Cyclic Loading Spectrum** ............................ 1
   1.1 Introduction ................................................................. 1
   1.2 Cyclic Loading Spectrum .................................................. 3
   1.3 References ................................................................. 7
   1.4 Exercises ................................................................. 7

2  **Reliability of a Component under Cyclic Load** .................... 9
   2.1 Introduction ................................................................. 9
   2.2 Fatigue Damage Mechanism ............................................. 9
   2.3 Fatigue Test, S-N Curve, and Material Endurance Limit .................. 11
   2.4 The Marin Modification Factors ........................................ 15
   2.5 The Effect of Mean Stress .............................................. 18
   2.6 The Fatigue Stress Concentration Factor ................................ 20
   2.7 Reliability of a Component with an Infinite Life .................. 22
   2.8 Reliability of a Component by the P-S-N Curves Approach .......... 29
      2.8.1 The Material P-S-N Curves ........................................ 29
      2.8.2 The Component P-S-N Curves ..................................... 37
      2.8.3 Reliability of a Component under Model #1 Cyclic Loading Spectrum .................................................. 41
      2.8.4 Reliability of a Component under Model #2 Cyclic Loading Spectrum .................................................. 47
      2.8.5 Reliability of a Component under Model #3 Cyclic Loading Spectrum .................................................. 48
      2.8.6 Reliability of a Component under Model #4 Cyclic Loading Spectrum .................................................. 52
      2.8.7 Reliability of a Component under Model #5 Cyclic Loading Spectrum .................................................. 55
      2.8.8 Reliability of a Component under Model #6 Cyclic Loading Spectrum .................................................. 64
Reliability-Based Mechanical Design consists of two separate books: Volume 1: Component under Static Load and Volume 2: Component under Cyclic Load and Dimension Design with Required Reliability.

Volume 1 consists of four chapters and Appendix A. They are:

• Chapter 1: Introduction to Reliability in Mechanical Design;
• Chapter 2: Fundamental Reliability Mathematics;
• Chapter 3: Computational Methods for the Reliability of a Component;
• Chapter 4: Reliability of a Component under Static load; and
• Appendix A: Samples of MATLAB Programs.

Volume 2 consists of three chapters and two appendixes. They are:

• Chapter 1: Introduction and Cyclic Loading Spectrum;
• Chapter 2: Reliability of a Component under Cyclic Load;
• Chapter 3: The Dimension of a Component with Required Reliability;
• Appendix A: Three Computational Methods for the Reliability of a Component; and
• Appendix B: Samples of MATLAB Programs.

The first book discusses fundamental concepts for implementing reliability in mechanical design and the reliability of a component under static load. The second book presents more advanced topics, including the reliability of a component under cyclic load and the dimension design with required reliability.

This is Reliability-based Mechanical Design, Volume 2: Component under Cyclic Load and Dimension Design with Required Reliability. It is recommended that Volume 1 should be read first before Volume 2 because it provides fundamental concepts and computational methods for implementing reliability in mechanical design and the reliability of a component under static load.

This book presents how to determine reliability, and quantitively predict the failure percentage of a component under cyclic load. This book also presents how to design component dimension with required reliability for a component under static load or cyclic load. Therefore,
the component will be reliable during its service under the specified load because it has been
designed with required reliability.

This book is based on the author’s recent research and a series of lecture notes of an elective
course for senior mechanical students. This book is written as a textbook for senior mechanical
students. Every topic is discussed in sufficient detail and demonstrated by many examples so
that students or design engineers can readily use them in mechanical design. At the end of each
chapter, there is a wide selection of exercise. This book can also be used for a graduate student
course and a reference book for design engineers.

This book consists of three chapters and two appendixes. A concise summary of each
chapter are as follows.

• Chapter 1: Introduction and Cyclic Load Spectrum
  This chapter presents a systematic description of a cyclic load. Six models of cyclic loading
  spectrum will be presented and can be used to describe any type of cyclic load.

• Chapter 2: Reliability of a Component under a Cyclic Load
  This chapter describes how to establish the limit state function of a component under a
  cyclic load, and then how to determine the reliability of a component under such cyclic
  load. The book presents two fatigue theories to calculate the reliability of a component
  under cyclic load. The first theory is the P-S-N (Probability-Stress-Number of cycles) curve
  approach. The second theory is the probabilistic fatigue damage model (the K-D model).
  Five typical component cases under cyclic load presented in this chapter include bar under
  cyclic axial load, pin under cyclic direct shearing, shaft under cyclic torsion, beam under
  cyclic bending, and a rotating shaft under cyclic combined loads.

• Chapter 3: The Dimension of a Component with Required Reliability
  This chapter presents how to design the dimension of a component with required reliability
  under static load or cyclic load. For the dimension of a component under cyclic load, the
  second fatigue theory, that is, the K-D model is mainly used. Five typical component
  dimension design with required reliability presented in this chapter include bar under axial
  load or cyclic axial load, pin under direct shearing or cyclic direct shearing, shaft under
torsion or cyclic torsion, beam under bending or cyclic bending, and a component under
  combined loads or cyclic combined loads.

• Appendix A: Computational Methods for Calculating the Reliability of a Component
  Appendix A concisely describes the procedures of the Hasoder-Lind (H-L) method, the
  Rachwitz-Fiessler (R-F) method, and the Monte Carlo method for calculating the reliability
  of a component, which has been presented in details in the first book: Reliability-based
  Mechanical Design, Volume 1: Component under Static Load.

• Appendix B: Samples of six MATLAB Programs
Appendix B provides six MATLAB program as a reference, including three programs for the calculation of reliability and another three programs for dimension design with required reliability.

This book could not have been completed and published without lots of encouragement and help. First, I sincerely thank Mechanical Department Chairman and Professor Mickael Jackson at the Wentworth Institute of Technology, whose encouragement motivated me to open two technical elective courses about the reliability in mechanical engineering. Second, I sincerely thank Professors Anthony William Duva and Richard L. Roberts for reviewing some of the manuscripts. Third, I sincerely thank Morgan & Claypool Publishers and Executive Editor Paul Petralia for helping with this publication. Finally, I sincerely thank my lovely wife, Suyan Zou. Without her support, I could not have completed this book.

Xiaobin Le
October 2019
CHAPTER 1

Introduction and Cyclic Loading Spectrum

1.1 INTRODUCTION

Reliability-Based Mechanical Design consists of two separate books: Volume 1: Component under Static Load and Volume 2: Component under Cyclic Load and Dimension Design with Required Reliability.

This book is Volume 2. It is recommended that Volume 1 should be read first before Volume 2 since fundamental concepts of probability theory and their implementation in mechanical design, as well as construction of the limit state function of a component under load are discussed in detail. The following are some concise notes on topics previously discussed in Volume 1 but that will be frequently used in this book.

Fundamental reliability mathematics, which discusses the fundamental concepts and definitions of probabilistic theory, is discussed in detail in Chapter 2 of Volume 1 [1]. The purpose of this is to provide a foundation and basic understandings for implementing probability theory in reliability-based mechanical design.

Computational methods of the reliability of a component, which discuss several computational methods including the Hasodler–Lind (H-L) method, the Rachwitz–Fiessler (R-F) method, and the Monte Carlo method, are discussed in Chapter 3 of Volume 1 [1]. The concise description of their procedures and flowcharts are presented in Appendix A of this book.

In reliability-based mechanical design, component geometric dimensions, loads, and stress concentration factor on a component are typically treated as random variables. These are discussed in detail in Chapter 4 of Volume 1 [1]. Since these will be frequently used in this book, we concisely describe them here.

Component geometric dimension is a random variable because of its dimension tolerance. It is typically treated as a normally distributed random variable \( d \). According to the definition of dimension tolerance, the components’ dimension inside the dimension tolerance range \( (d + t_L, d + t_U) \) will be accepted. For a normal distribution, the probability of event \( (\mu_d - 4\sigma_d \leq d \leq \mu_d + 4\sigma_d) \) will be 99.9968%. This event can be used to represent the dimension tolerance range with a very small error (0.0032%). Therefore, the mean and standard deviation of a normally distributed dimension random variable \( d \) can be determined per Equa-
2 1. INTRODUCTION AND CYCLIC LOADING SPECTRUM


\[
\begin{align*}
\mu_d &= \frac{(d + t_L) + (d + t_U)}{2} = d + \frac{t_L + t_U}{2}, \\
\sigma_d &= \frac{(d + t_U) - (d + t_L)}{8} = \frac{t_U - t_L}{8},
\end{align*}
\]  

(1.1)

where \( \mu_d \) and \( \sigma_d \) are the mean and standard deviation of a normally distributed dimension \( d \).

The type of distribution and its corresponding distribution parameters of an external load will be calculated per the design specifications of a design case. If a load \( P \) such as concentrated force, concentrated moment, or torque, is expressed as a range of value such as \( (P_{\text{low}}, P_{\text{up}}) \), it could be treated as a normally distributed random variable. We can use the same reasoning and similar equation as Equation (1.1) to determine its mean and standard deviation:

\[
\begin{align*}
\mu_P &= \frac{(P_{\text{low}} + P_{\text{up}})}{2} \\
\sigma_P &= \frac{(P_{\text{up}} - P_{\text{low}})}{8},
\end{align*}
\]  

(1.2)

where \( \mu_P \) and \( \sigma_P \) are the mean and standard deviation of a normally distributed load \( P \).

The static stress concentration factor is a function of geometric shape and dimension. Since the geometric dimension is a random variable, the stress concentration factor is also a random variable and typically follows a normal distribution. We can use the following equations to determine the mean and the standard deviation of stress concentration factor:

\[
\begin{align*}
\gamma_K &= 0.05 \\
\mu_K &= K_{\text{table}} \\
\sigma_K &= \gamma_K \times \mu_K = 0.05 K_{\text{table}},
\end{align*}
\]  

(1.3)

where \( \gamma_K \) is the coefficient of variance of the static stress concentration factor. \( K_{\text{table}} \) is the stress concentration factor obtained from tables in current design handbooks or design books. \( \mu_K \) and \( \sigma_K \) are the mean and standard deviation of normally distributed static stress concentration factors.

This book consists of three chapters and two appendixes. The concise outlines of each chapter are as follows.

• Chapter 1: Introduction and Cyclic Loading Spectrum

This chapter explains the connection of this book with the first book: Reliability-Based Mechanical Design, Volume 1: Component under Static Load. Then, this chapter presents a systematic description of a cyclic load. Six models of cyclic loading spectrum will be presented and can be used to describe any type of cyclic load.

• Chapter 2: Reliability of a Component under a Cyclic Load
1.2 CYCLIC LOADING SPECTRUM

This chapter describes how to establish the limit state function of a component under a cyclic load, and then how to determine the reliability of a component under such cyclic load. The book presents two fatigue theories to calculate the reliability of a component under cyclic load. The first theory is the P-S-N (Probabilistic-Stress-Number of cycles) curve approach. The second theory is the probabilistic fatigue damage model (the K-D model). Five typical component cases under cyclic load presented in this chapter include bar under cyclic axial load, pin under cyclic direct shearing, shaft under cyclic torsion, beam under cyclic bending, and a rotating shaft under cyclic combined loads.

• Chapter 3: The Dimension of a Component with the Required Reliability
  This chapter presents how to design the dimension of a component with required reliability under static load or cyclic load. For the dimension of a component under cyclic load, the second fatigue theory, that is, the K-D model is mainly used. Five typical component dimension design with required reliability presented in this chapter include bar under static axial load or cyclic axial load, pin under static direct shearing or cyclic direct shearing, shaft under static torsion or cyclic torsion, beam under static bending or cyclic bending, and a component under combined static loads or cyclic combined loads.

• Appendix A: Computational Methods for Calculating the Reliability of a Component
  This appendix concisely describes the procedure of the H-L, R-F, and Monte Carlo methods for calculating the reliability of a component, which has been presented in detail in Volume 1 [1].

• Appendix B: Samples of Six MATLAB Programs
  This appendix provides six MATLAB programs as a reference, including three programs for the calculation of reliability and another three programs for dimension design with required reliability.

1.2 CYCLIC LOADING SPECTRUM

Mechanical devices or systems always have at least one moving component. Due to the repeated functions or stop-start process or mechanical vibration, mechanical components are typically subjected to a cyclic load. A schematic of a cyclic load is depicted in Figure 1.1. The maximum stress \( \sigma_{\text{max}} \) and the minimum stress \( \sigma_{\text{min}} \) of cyclic stress (loading) are the maximum and minimum values of the cyclic stresses, as shown in Figure 1.1. The mean stress \( \sigma_m \), the stress amplitude \( \sigma_a \), and the range of stress \( \sigma_r \) of the cyclic stress (loading) are defined as:

\[
\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \quad (1.4)
\]

\[
\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \quad (1.5)
\]
The stress ratio $S_r$ is defined as the ratio of the minimum stress $\sigma_{\text{min}}$ to the maximum stress $\sigma_{\text{max}}$, that is,

$$S_r = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}.$$  \hspace{1cm} (1.7)

A cyclic stress curve can be treated as a wave. One complete of the wave such as the minimum point to the adjacent minimum point, or the maximum point to the adjacent maximum point is one cycle of the cyclic stress, as shown in Figure 1.1.

![Figure 1.1: A schematic of cyclic stress with a constant stress amplitude.](image)

The magnitude of cyclic stress can be fully defined by any two out of these six variables $\sigma_{\text{max}}$, $\sigma_{\text{min}}$, $\sigma_m$, $\sigma_a$, $\sigma_r$, and $S_r$. The duration of the cyclic stress will be defined by the number of cycles of the cyclic loading. One special cyclic stress that has a zero mean stress is called the fully (completely) reversed cyclic stress, as shown in Figure 1.2. For a fully reversed cyclic stress, it has: $\sigma_{\text{max}} = -\sigma_{\text{min}}$, and the stress ratio $S_r = -1$. This type of cyclic stress is a special case because lots of fatigue strength data are based on fatigue tests under a fully reversed cyclic stress.

**Example 1.1**

Cyclic stress has a stress amplitude $\sigma_a = 10$ ksi and the stress ratio $S_r = 0.5$. Calculate the mean stress $\sigma_m$, the maximum stresses $\sigma_{\text{max}}$, the minimum stresses $\sigma_{\text{min}}$, and the range of stress $\sigma_r$ of this cyclic stress.

**Solution:**

Based on Equations (1.5) and (1.7), we have:

$$\sigma_a = 10 = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \quad \text{(a)}$$

$$S_r = 0.5 = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}.$$ \hspace{1cm} (b)
1.2. CYCLIC LOADING SPECTRUM

Figure 1.2: A fully reversed cyclic stress.

From Equations (a) and (b), we have:

\[ \sigma_{\text{max}} = 40 \text{ (ksi)}, \quad \sigma_{\text{min}} = 20 \text{ (ksi)}. \]  

(c)

Based on Equations (1.4) and (1.6) by using information from Equation (c), we have:

\[ \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} = \frac{40 + 20}{2} = 30 \text{ (ksi)} \]
\[ \sigma_r = \sigma_{\text{max}} - \sigma_{\text{min}} = 40 - 20 = 20 \text{ (ksi)}. \]

There is a lot of different cyclic loads. The cyclic loading spectrum refers to a description of cyclic stress (loading) levels vs. corresponding cycle numbers. Generally, three parameters, including stress amplitude, mean stress, and the number of cycles of the cyclic loading, are used to describe a cyclic loading spectrum fully. However, for fatigue design, a non-zero mean cyclic loading is typically converted into a fully reversed cyclic loading with an equivalent stress amplitude, which will be discussed in detail in Chapter 2. So, two parameters including the fully reversed stress amplitude \( \sigma_a \) and the cycle number \( n_L \) of cyclic loading are typically used to describe a cyclic loading spectrum for the reliability calculation of a component under cyclic load. Both stress amplitude \( \sigma_a \) and the cycle number \( n_L \) can be a constant (deterministic value) or several constant or random variable. With the reasonable combinations of variations of stress amplitude \( \sigma_a \) and the cycle number \( n_L \), the systematic description of cyclic loading spectrum will include the following six models. These six cyclic loading spectrum models [2] can describe any cyclic loading spectrum.

Model #1: A constant stress amplitude of cyclic loading with a constant cycle number.

Model #1 is the simplest cyclic loading spectrum for fatigue design. For example, the component under design is subjected to a constant cyclic stress amplitude \( \sigma_a = 15 \text{ (ksi)} \) with a given cycle number \( n_L = 5 \times 10^4 \text{ (cycles)}. \)
6 1. INTRODUCTION AND CYCLIC LOADING SPECTRUM

**Model #2:** A constant stress amplitude of cyclic loading with a distributed cycle number.

Model #2 is a typical cyclic loading spectrum for a component with a single steady function, that is, a constant cyclic stress amplitude. However, the cycle number \(n_L\) of the cyclic loading is treated as a random variable and is described by a probabilistic distribution function. How can this be? It is because, in the reliability-based mechanical design, the components under design are a batch of “identical” components in service. Each component during its service life had one value of the number of cycles. All of those can be used to determine the distribution function of the cycle number. For example, the component under design is subjected to a constant stress amplitude of a fully reversed cyclic stress \(\sigma_a = 15\) (ksi) with a normally distributed cycle number \(n_L\), which has a mean \(\mu_{nL} = 3 \times 10^5\) (cycles), and a standard deviation \(\sigma_{nL} = 3500\) (cycles), that is, \(n_L \sim N(3 \times 10^5, 3500)\). Here, the expression \(X \sim N(\mu_x, \sigma_x)\) means that the random variable \(X\) is a normal distribution with a mean \(\mu_x\) and a standard deviation \(\sigma_x\).

**Model #3:** A given fatigue life (cycle number) with a distributed amplitude of a cyclic loading.

Model #3 is a typical cyclic loading spectrum for the component with specified service life, but the fully reversed stress amplitude \(\sigma_a\) varies and can be treated as a random variable, and is described by a probabilistic distribution function. For example, the component under design with a cycle number \(n_L = 8 \times 10^4\) (cycles) is subjected to a fully reversed cyclic stress. The stress amplitude \(\sigma_a\) of this cyclic loading follows the uniform distribution between 25 (ksi) and 35 (ksi).

**Model #4:** Multiple constant amplitudes of cyclic loadings with multiple constant cycle numbers.

Typically, model #4 could be used to describe the cyclic loading spectrum of a machine with several distinguished functions or actions. For example, the cyclic loading spectrum of the component under design is:

- Cyclic stress level 1: \(\sigma_{a1} = 20\) (ksi), \(n_{L1} = 260,000\) (cycles)
- Cyclic stress level 2: \(\sigma_{a2} = 30\) (ksi), \(n_{L2} = 50,000\) (cycles).

**Model #5:** Multiple constant stress amplitudes of cyclic loadings with multiple distributed cycle numbers.

Model #5 is a common cyclic loading spectrum and can be used to describe many loading conditions for machines with several distinguished functions. For example, the cyclic loading spectrum of the component under design is:

- Cyclic stress level 1: \(\sigma_{a1} = 20\) (ksi); \(n_{L1} \sim N(260,000, 10,000)\)
- Cyclic stress level 2: \(\sigma_{a2} = 25\) (ksi); \(\ln(n_{L2}) \sim N(8.425, 0.136)\).
Model #6: Multiple distributed stress amplitudes of cyclic loading levels with multiple given cycle numbers.

Model #6 is also a common cyclic loading spectrum and can be used to describe many loading conditions for the machines, the service of which are pre-scheduled. For example, the cyclic loading spectrum of the component under design is:

Cyclic stress level 1: \( n_{L1} = 260,000 \) (cycles); \( \sigma_{a1} = N(20,000, 1890) \)
Cyclic stress level 2: \( n_{L2} = 5000 \) (cycles); \( \ln(\sigma_{a2}) = N(3.25, 0.108) \).

Any cyclic loading for fatigue design can be described by one of the above six fatigue cyclic loading spectrum models. Thus, they are a systematic description of cyclic loading spectrum.

1.3 REFERENCES


1.4 EXERCISES

1.1. Cyclic stress has a maximum stress \( \sigma_{\text{max}} = 60.25 \) (ksi) and a minimum stress \( \sigma_{\text{min}} = -9.32 \) (ksi). Calculate its mean stress, stress amplitude, and stress ratio.

1.2. Cyclic stress has a constant mean \( \sigma_{m} = 15.72 \) (ksi) and a stress amplitude \( \sigma_{a} = 25.39 \) (ksi). Calculate its maximum stress and the minimum stress of this cyclic stress.

1.3. What is cyclic loading spectrum? Describe one example.

1.4. What causes cyclic stress? Use two examples to explain the lists.

1.5. Describe an example in which model #1 cyclic loading spectrum can be used to describe its cyclic stress.

1.6. Describe an example in which model #2 cyclic loading spectrum can be used to describe its cyclic stress.

1.7. Describe an example in which model #3 cyclic loading spectrum can be used to describe its cyclic stress.

1.8. Describe an example in which model #4 cyclic loading spectrum can be used to describe its cyclic stress.
1. INTRODUCTION AND CYCLIC LOADING SPECTRUM

1.9. Describe an example in which model #5 cyclic loading spectrum can be used to describe its cyclic stress.

1.10. Describe an example in which model #6 cyclic loading spectrum can be used to describe its cyclic stress.
CHAPTER 2

Reliability of a Component under Cyclic Load

2.1 INTRODUCTION

Metal components under repeated loadings, that is, cyclic load, can fracture even though the component’s maximum nominal stress is far less than ultimate material strength or yield strength. This type of failure is fatigue failure. In industries, more than 90% of metal component failure is due to fatigue failure.

A fatigue issue with the number of cycles at failure between 1 and $10^3$ is generally classified as low-cycle fatigue. When the number of cycles at failure is more than $10^3$, it is high-cycle fatigue. This chapter will only focus on high-cycle fatigue, which is the typical case for mechanical component design in industries.

The component design, that is, determination of component geometric dimension under cyclic loading with the specified reliability will be discussed in Chapter 3. This chapter will show and explain how to calculate the reliability of a component under cyclic loading. The reliability calculation of a component with an infinite life will be discussed in Section 2.7. Two different probabilistic fatigue theories will be used to conduct the reliability calculation of a component with a finite fatigue life. One theory is the P-S-N curves approach, which will be discussed and explained in Section 2.8. Another theory is the probabilistic fatigue damage theory, which includes fatigue strength index $K$ and the fatigue damage index $D$. This probabilistic fatigue damage theory can be called as the K-D model, which will be discussed and explained in Section 2.9.

This chapter will present and discuss different methods to determine the reliability of a component under any cyclic load with plenty of examples.

2.2 FATIGUE DAMAGE MECHANISM

Fatigue phenomena were first discovered and studied during the 19th century with the arrival of machines and freight vehicles during the industrial revolution [1]. Fatigue is defined as “failure under a repeated or varying loading, which never reaches a level sufficient to cause failure in a single application.”

Fatigue failure of a metal component under cyclic loading is a complicated phenomenon, and only partially understood [2]. However, we have a fundamental understanding of fatigue
failure or fatigue damage. Fatigue damage is the weakening of a metal material due to a gradually crack propagation of inherent existing microscopic cracks or defect inside or on the surface of the metal component under repeated cyclic loading. Without a crack inside or on the surface of a metal component, there is no fatigue. If the magnitude of cyclic loading is not big enough to generate a crack propagation, there will be no fatigue.

The fatigue damage mechanism can be typically described by the following four stages. Let use a microscopic crack on the surface to explain and demonstrate these. Figure 2.1 shows a magnified microscopic crack on the surface of a component under a fully reversed cyclic bending moment. In this example, let us assume that the nominal normal stress due to the bending stress is 20 ksi, and the material yield strength is 60 ksi.

1. Crack initiation. There are always lots of randomly distributed defects inside a component such as voids and dislocations and on the surface of a component such as manufacturing scratches [3]. A fatigue crack will typically initiate at a microscopic crack or defect inside or on the surface of a component. As shown in Figure 2.1a, the bending moment tries to open the microscopic crack “A”, which will have very high stress because of the sharp tip of the microscopic crack “A”. Let us assume that the stress concentration factor in this situation is 3.5. So, the maximum stress at the tip of the microscopic crack “A” could be $3.5 \times 20 = 70$ ksi and will be larger than the material yield strength. The material at the tip of the crack “A” will start yielding and has some plastic deformation, as shown in Figure 2.1b. When the material at the tip is yielding, the sharp tip of the crack “A” will become a dull tip, and the stress concentration factor decreases. Let assume that the stress concentration factor becomes 2.5. Now the maximum stress at the dull tip of the crack “A” is $2.5 \times 20 = 50$ ksi. It is less than the material yield strength. Therefore, the yielding at the tip of the crack “A” will stop. If external loading is not a cyclic loading, the effect of the microscopic crack “A” on the component is negligible and can be ignored because the plastic deformation at the tip of the crack “A” is in microscopic level.

2. Crack propagation. When the component is under a fully reversed cyclic loading, the reversed bending moment now tries to close the microscopic crack “A”, as shown in Figure 2.1c. It is well known that if the dull tip of the microscopic crack “A” undergoes “strain hardening” due to the yielding [3], the material around the dull tip area will be brittle. After one time or a few times of such “open” and “close” actions, a new microscopic crack “B” will be generated beneath the crack “A”. This result is a crack propagation, as shown in Figure 2.1c. When the component is under a fully reversed cyclic loading, the microscopic crack “B” will be opened again as shown in Figure 2.1d, which is the same situation as shown in Figure 2.1a. In this stage, the crack could continue to grow as a result of continuously applied cyclic loading.

3. Fracture due to static loading. Eventually, a crack will reach a critical size and the effective cross-sectional area of the component is so reduced that the actual stress on the effective
area under a normal service cyclic load will be larger than the ultimate material strength, causing the component to rupture due to static loading.

4. Fatigue damage is irreversible and will be gradually accumulated. The fatigue damage is due to crack propagation. When the cyclic loading stops, the propagated microscopic cracks still exist. Therefore, the fatigue damage is irreversible and will be gradually accumulated on the continuous cyclic loading.

2.3 FATIGUE TEST, S-N CURVE, AND MATERIAL ENDURANCE LIMIT

A cyclic load applied on a component can be any type of cyclic load and can be described by six models of cyclic loading spectrum [4], which has been described in Section 1.2. But, lots of material fatigue strength data is typically obtained from a stress-life method. In the stress-life method, a specimen is subjected to cyclic stress with a constant stress amplitude until it fractures and fails. There are many different types of fatigue specimen and fatigue test procedures. Fatigue test specimen will be designed and manufactured according to corresponding fatigue standards such as ASTM standards, and the test procedure will also follow the procedure defined by corresponding fatigue test standards. The cyclic stress for a fatigue test could be cyclic bending stress, cyclic axial stress, or cyclic shear (torsion) stress. The cyclic stress in a stress-life method is typically a fully reversed cyclic stress, that is, a constant stress amplitude with zero-mean stress.

The main reasons for this are as follows. (1) Lots of fatigue test data are from rotating bending fatigue test, in which the cyclic stress is a fully reversed cyclic stress. (2) In fatigue theory for fatigue design, non-zero mean cyclic stress will typically be converted into fully reversed cyclic stress with an equivalent stress amplitude by including the effect of mean stress. (3) Even though fatigue tests are under cyclic stress with non-zero mean stress, it might be still presented as fatigue test data with an equivalently fully reversed cyclic stress for the purpose that the fatigue test data can be used for fatigue design. In the following, we will assume that cyclic stress in the stress-life method is a fully reversed cyclic stress.

In a stress-life method with a fatigue test specimen under a fully reversed cyclic stress, test results are the stress amplitude $S_f'$ and the number of cycles at the failure $N$. Both $S_f'$ and $N$ is material fatigue strength data. This stress amplitude $S_f'$ in a fatigue test is called as the
2. RELIABILITY OF A COMPONENT UNDER CYCLIC LOAD

Material fatigue strength at the given number of cycles $n_L = N$. The physical meaning of this fatigue strength $S'_f$ is that when the number of cycles of a fully reversed cyclic stress is $n_L = N$, the fatigue specimen will fail if the stress amplitude $\sigma_a$ of a fully reversed cyclic stress is more than $S'_f$. In other words, the maximum stress amplitude of a fully reversed cyclic stress cannot exceed $S'_f$ to avoid a fatigue failure when the service life is specified as $n = N$. The number of cycles at failure $N$ in the fatigue test is called as the material fatigue life at this specified stress level $\sigma_a = S'_f$ of a fully reversed cyclic stress amplitude. The physical meaning of the material fatigue life $N$ is that if the fatigue test specimen is under a fully reversed cyclic stress with a stress amplitude $\sigma_a = S'_f$, the fatigue specimen will fail when the service life $n_L$ of such cyclic stress is more than $N$. Therefore, the fatigue test results $(S'_f, N)$ is a pair of fatigue strength data in a fatigue test.

After fatigue tests on the fatigue specimen of the same material are continuously conducted at different stress amplitudes (stress levels), a group data $(S'_f, N)$ will be collected and can be depicted as an S-N curve, as shown in Figure 2.2. The S-N curve is typically plotted in a Cartesian coordinate with a log-log scale.

![S-N curve](image)

**Figure 2.2:** Schematic of an S-N curve.

In Figure 2.2, the small dots are a pair of fatigue test data. There are three different fatigue regimes, as shown in Figure 2.2. The fatigue failure form $N = 1$ to $N = 1000$ (cycles) is generally classified as low-cycle fatigue. In low-cycle fatigue, the stress level at the number of cycles at failure $N = 1$ is the ultimate material strength $S_{ut}$. For low-cycle fatigue, the test method is typically through strain-life method or linear-elastic fracture mechanics method, in which, the strain or the crack growth will be controlled or measured. This low-cycle fatigue is not the concerned topics of this book.

The fatigue failure with $N > 1000$ (cycles) is generally called as high-cycle fatigue. The high-cycle fatigue will be the focus of this book and is the typical case for most of the fatigue design in the industry.
Material endurance limit: For some materials like steels as shown in Figure 2.2, there is a value below which fatigue specimen will not fail with a very large number of cycles such as more than $10^6$ (cycles). Material endurance limit $S'_e$ is usually defined as the maximum fully reversed stress amplitude that a material can withstand infinitely without fracture. For some materials, the fatigue strength $S'_f$ at the fatigue life $N = 10^6$ (cycles) is named as material endurance limit $S'_e$.

In high-cycle fatigue, a fatigue life between $N = 10^3$ (cycles) and $N = 10^6$ (cycles) is defined as a finite-life region, and a fatigue life $N \geq 10^6$ (cycles) is defined as an infinite life.

In a finite-life region, when there are fatigue tests on at least three different stress amplitude levels of fully reversed cyclic stress, the average fatigue life $N$ at the same fatigue stress level vs. the fatigue strength $S'_f$ in a log-log scale coordinate system can be typically simplified as a linear line, as shown in Figure 2.2. The material fatigue strength $S'_f$ and the fatigue life $N$ on this linear line has the following relationship:

$$N(S'_f)^m = \text{Constant},$$  \hspace{1cm} (2.1)

where $S'_f$ and $N$ are the material fatigue strength and the corresponding fatigue life on the simplified linear line. $m$ is the slope of the traditional S-N curve and is a material mechanical property determined by fatigue test data. $m$ can be determined through the linear least-squares regression by using the fatigue test results:

$$m = \frac{I \left[ \sum_i (\ln \sigma_{ai} \cdot \ln N_i) \right] - \left( \sum_i \ln \sigma_{ai} \right) \left( \sum_i \ln N_i \right)}{I \left[ \sum_i (\ln \sigma_{ai})^2 \right] - (\sum_i \ln \sigma_{ai})^2},$$  \hspace{1cm} (2.2)

where $I$ is the number of different stress amplitude levels $\sigma_{ai}$ for fatigue tests; $\sigma_{ai}$ is the $i$th stress amplitude level in fatigue tests; $\ln N_i$ is the average fatigue life in a log-scale at the fatigue test level $\sigma_{ai}$. If there are a total $J$ fatigue tests at the fatigue test stress level $\sigma_{ai}$, the $\ln N_i$ will be:

$$\ln N_i = \frac{\sum_j \ln N_{ij}}{J},$$  \hspace{1cm} (2.3)

where $N_{ij}$ is the number of cycles at the failure of the $j$th fatigue test under the fatigue test stress level $\sigma_{ai}$.

When there are only a few fatigue tests for the S-N curve, Equation (2.1) is the design equation for the traditional fatigue design approach. The case with plenty of fatigue tests will be discussed in Sections 2.8 and 2.9 and will be the focus of this book.

Example 2.1
Fatigue tests of steel specimens under a fully reversed cyclic bending stress are listed in Table 2.1. Determine the material property $m$ on a log-log scale.
### 2. RELIABILITY OF A COMPONENT UNDER CYCLIC LOAD

#### Table 2.1: A group of fatigue test data

<table>
<thead>
<tr>
<th>Stress Amplitude $\sigma_a$ (Mpa)</th>
<th>Sample Size</th>
<th>Fatigue Life $N$ (cycles) $\times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>392.40</td>
<td>4</td>
<td>34, 42, 43, 48</td>
</tr>
<tr>
<td>372.78</td>
<td>6</td>
<td>36, 47, 48, 53, 62, 65</td>
</tr>
<tr>
<td>353.16</td>
<td>6</td>
<td>60, 70, 77, 84, 89, 116</td>
</tr>
<tr>
<td>333.54</td>
<td>4</td>
<td>111, 114, 145, 197</td>
</tr>
<tr>
<td>313.92</td>
<td>5</td>
<td>171, 253, 254, 301, 309</td>
</tr>
</tbody>
</table>

**Solution:**

In this group of fatigue tests, there are five different stress amplitude levels. So, $I = 5$. In each fatigue stress levels, the fatigue tests are repeated for several times. We will use Equation (2.3) to calculate the average fatigue life in a log-scale at each stress amplitude level. For example, there are six repeated fatigue tests on the third stress level $\sigma_a = 353.16$ (Mpa), the average fatigue life in a log scale $\log N_3$ will be:

$$
\ln N_3 = \frac{\ln (60000) + \ln (70000) + \ln (77000) + \ln (84000) + \ln (89000) + \ln (116000)}{6}
$$

$$
= 11.30523.
$$

The stress amplitudes and corresponding average fatigue life on a log-log scale for this example are listed in Table 2.2.

By using the data from Table 2.2 with Equation (2.2), the material property $m$ is:

$$
m = 8.303.
$$

#### Table 2.2: Average log stress amplitudes and fatigue life

<table>
<thead>
<tr>
<th>Stress level #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln \sigma_{ai}$</td>
<td>5.972282</td>
<td>5.920989</td>
<td>5.866922</td>
<td>5.809763</td>
<td>5.749138</td>
</tr>
<tr>
<td>$\ln N_i$</td>
<td>10.63186</td>
<td>10.8372</td>
<td>11.30104</td>
<td>11.83417</td>
<td>12.43832</td>
</tr>
</tbody>
</table>

For fatigue design with an infinite fatigue life, the material endurance limit $S'_e$ is one fatigue strength data for the material. When there is a lack of fatigue test data for material endurance limit $S'_e$, it can be estimated by using the ultimate strength of the same material [5].
2.4 THE MARIN MODIFICATION FACTORS

Material fatigue strength data are typically obtained from fatigue tests on standard rotating-beam bending specimen under fully reversed cyclic bending stress. The fatigue specimens are designed according to the fatigue test standards. For example, the rotating-beam bending stress specimen has a polish surface finish and a curved cylindrical shape with a smallest diameter 0.300" in the middle of the specimen. The fatigue tests are typically under a fully reversed cyclic bending stress at the room temperature. For fatigue design, a component under consideration

For bending cyclic loading:

For steel, \[ S'_{\text{e}} = \begin{cases} 
0.5S_{\text{ut}} & S_{\text{ut}} < 1400 \text{ Mpa (200 ksi)} \\
700 \text{ (Map) (100 ksi)} & S_{\text{ut}} \geq 1400 \text{ Mpa (200 ksi)}. 
\end{cases} \] (2.4)

For iron, \[ S'_{\text{e}} = \begin{cases} 
0.4S_{\text{ut}} & S_{\text{ut}} < 400 \text{ Mpa (60 ksi)} \\
160 \text{ (Map) (24 ksi)} & S_{\text{ut}} \geq 400 \text{ Mpa (60 ksi)}. 
\end{cases} \] (2.5)

For aluminum, \[ S'_{\text{e}} = \begin{cases} 
0.4S_{\text{ut}} & S_{\text{ut}} < 330 \text{ Mpa (48 ksi)} \\
130 \text{ (Map) (19 ksi)} & S_{\text{ut}} \geq 330 \text{ Mpa (48 ksi)}. 
\end{cases} \] (2.6)

For copper alloy, \[ S'_{\text{e}} = \begin{cases} 
0.4S_{\text{ut}} & S_{\text{ut}} < 280 \text{ Mpa (40 ksi)} \\
100 \text{ (Map) (14 ksi)} & S_{\text{ut}} \geq 280 \text{ Mpa (40 ksi)}. 
\end{cases} \] (2.7)

For axial cyclic loading:

For steel, \[ S'_{\text{e}} = 0.45S_{\text{ut}}. \] (2.8)

For cyclic torsional loading:

For steel, \[ S'_{\text{e}} = 0.29S_{\text{ut}}. \] (2.9)

For iron, \[ S'_{\text{e}} = 0.32S_{\text{ut}}. \] (2.10)

For copper alloy, \[ S'_{\text{e}} = 0.22S_{\text{ut}}. \] (2.11)
will have a different surface finish, different dimension, and different types of cyclic loading. Different surface finish will have quite different initial defects or cracks on the surfaces. A component with a bigger dimension means that it will have a much higher likelihood of more initial defects inside components. The maximum stress' area of a component due to bending, torsion, and axial loading are quite different. For a component under bending, the maximum stress will happen on the uppermost and lowermost layers. For a component under torsion, the maximum stress will appear on the outer surface. However, a component under axial loading, the maximum stress will appear on the whole cross-section. Therefore, component fatigue strength will be different from the material fatigue strength obtained from fatigue specimen tests. This difference of fatigue strength between fatigue test specimen and a component is typically considered by several Marin modification factors [2, 6, 7]. Those modifications on the material fatigue test data are based on the rotating-beam bending fatigue test under a fully reversed cyclic bending stress. The following equation can calculate component endurance limit \( S_e \) at the critical section:

\[
S_e = k_a k_b k_c S'_e.
\]  

(2.12)

where \( S'_e \) is the material endurance limit obtained from fatigue test on the fatigue test specimen. \( k_a \) is the surface finish modification factor. \( k_b \) is the size modification factor. \( k_c \) is the loading modification factor. A mechanical component might have several different component endurance limits at different critical section due to the different size modification factors.

Component fatigue strength \( S_f \) at a given fatigue life \( N \) can be obtained through the following equation:

\[
S_f = k_a k_b k_c S'_f.
\]  

(2.13)

where \( S'_f \) is material fatigue strength at the fatigue life \( N \), which is the number of cycles at failure in the fatigue test. For fatigue design, the component fatigue strength \( S_f \) is not one value and will have a different value at different given fatigue life \( N \). The rests in Equation (2.13) are the same as those in Equation (2.12).

The surface finish modification factor \( k_a \) can be treated as a normally distributed random variable. Its mean \( \mu_{k_a} \) [7] will be calculated by the following equations:

\[
\mu_{k_a} = \begin{cases} 
16.45 \left( S_{ut} \right)^{-0.7427} & \text{For hot-rolled component} \\
39.9 \left( S_{ut} \right)^{-0.995} & \text{For as-forged component} \\
2.7 \left( S_{ut} \right)^{-0.2653} & \text{For machined surface component} \\
1.34 \left( S_{ut} \right)^{-0.0848} & \text{For ground surface component}
\end{cases}
\]  

(2.14)

where \( S_{ut} \) is material ultimate tensile strength in the unit of ksi.
2.4. THE MARIN MODIFICATION FACTORS

Its standard deviation $\sigma_{ka}$ will be calculated by using an estimated coefficient of variance $\gamma_{ka}$ [7] of the surface finish modification factor $k_a$ by the following equations:

$$\gamma_{ka} = \begin{cases} 
0.098 & \text{For hot-rolled component} \\
0.078 & \text{For as-forged component} \\
0.06 & \text{For machined surface component} \\
0.131 & \text{For ground surface component}
\end{cases}$$

(2.15)

$$\sigma_{ka} = \gamma_{ka} \times \mu_{ka}.$$  

(2.16)

The size modification factor $k_b$ will be treated as a deterministic and can be calculated by the following equation [7]:

$$k_b = \begin{cases} 
\left( \frac{d}{0.3} \right)^{-0.1133} & \text{For bending or torsion load with } 0.11'' \leq d \leq 2'' \\
1 & \text{For axial load}
\end{cases}$$

(2.17)

where $d$ is the diameter (or equivalent diameter) of the component in the unit of inch at the critical section.

The load modification factor $k_c$ can be treated as a normally distributed random variable. Its mean $\mu_{kc}$ can be calculated by the following equation [7]:

$$\mu_{kc} = \begin{cases} 
1 & \text{For bending load} \\
0.774 & \text{For axial load} \\
0.583 & \text{For torsional load}
\end{cases}$$

(2.18)

Its standard deviation $\sigma_{kc}$ will be calculated by using an estimated coefficient of variance $\gamma_{kc}$ [7] of the load modification factor $k_c$ by the following equations:

$$\gamma_{kc} = \begin{cases} 
0 & \text{For bending load} \\
0.163 & \text{For axial load} \\
0.123 & \text{For torsional load}
\end{cases}$$

(2.19)

$$\sigma_{kc} = \gamma_{kc} \times \mu_{kc}.$$  

(2.20)

In Equation (2.19), the coefficient of variance $\gamma_{kc}$ for bending loads is zero, and the mean value $\mu_{kc}$ is 1. This result is because the fatigue strength test data comes from cyclic bending loading.

**Example 2.2**

A machined bar with a diameter $1.5''$ is subjected to a cyclic torsion loading. Its ultimate material strength is 61.5 ksi. If the fatigue test data are obtained from rotating-beam specimen under
fully reversed cyclic bending stress, determine the surface finish modification factor \( k_a \), the size modification factor \( k_b \), and the load modification factor \( k_c \).

**Solution:**

The surface finish modification factor \( k_a \) will be treated as a normally distributed random variable. The mean \( \mu_{k_a} \) of the surface finish modification factor \( k_a \) per Equation (2.14) is:

\[
\mu_{k_a} = 2.7 (S_u)^{-0.2653} = 2.7(61.5)^{-0.2653} = 0.9053.
\]

The coefficient of variance of \( k_a \) per Equation (2.15) is:

\[
\gamma_{k_a} = 0.06.
\]

The standard deviation of \( k_a \) per Equation (2.16) is

\[
\sigma_{k_a} = \gamma_{k_a} \times \mu_{k_a} = 0.06 \times 0.9053 = 0.0543.
\]

The size modification factor \( k_b \) will be treated as a deterministic per Equation (2.17) is:

\[
k_b = \left( \frac{d}{0.3} \right)^{-0.1133} = \left( \frac{1.5}{0.3} \right)^{-0.1133} = 0.8333.
\]

The load modification factor \( k_c \) is treated as a normal distributed random variable. Its mean \( \mu_{k_c} \) per Equation (2.18) is:

\[
\mu_{k_c} = 0.583.
\]

The coefficient of variance of \( \gamma_{k_c} \) per Equation (2.19) is:

\[
\gamma_{k_c} = 0.123.
\]

The standard deviation of \( k_c \) per Equation (2.20) is

\[
\sigma_{k_c} = \gamma_{k_c} \times \mu_{k_c} = 0.123 \times 0.583 = 0.0717.
\]

2.5 **THE EFFECT OF MEAN STRESS**

Fatigue strength data is typically from fatigue tests under a fully reversed cyclic stress. Even when a fatigue test is under a non-zero-mean cyclic stress, it is typically presented as a fatigue strength data with an equivalent stress amplitude of a fully reversed cyclic stress. This approach is simply because general cyclic loading for mechanical component fatigue design might be any non-zero-mean stress cyclic stress. There are many fatigue theories such as Soderberg approach, Modified Goodman approach, Gerber approach, and ASME-Elliptic approach for considering
the effect of mean stress [2, 5]. This book will use the Modified Goodman approach to consider the effect of mean stress in cyclic stress through the following equation:

$$\sigma_{a-eq} = \begin{cases} \sigma_a \left( \frac{S_u}{S_u - \sigma_m} \right) & \text{when } \sigma_m \geq 0 \\ \sigma_a & \text{when } \sigma_m < 0 \end{cases}$$  \hspace{1cm} (2.21)$$

where $\sigma_a$ and $\sigma_m$ are the stress amplitude and the mean stress of cyclic stress. In Equation (2.21), $S_u$ is the ultimate material strength as a deterministic value, which will be equal to the average value of the ultimate material strength. $\sigma_{a-eq}$ is the equivalent stress amplitude of a fully reversed cyclic stress. For cyclic stress with negative mean stress, the equivalent stress amplitude will be equal to the stress amplitude of the cyclic stress with negative mean stress because the compressed mean stress will help to stop the crack propagation. Therefore, the Modified Goodman approach is more conservative.

**Example 2.3**

A component is subjected to cyclic stress with a mean stress $\sigma_m = 5$ (ksi) and a stress amplitude $\sigma_a = 17$ (ksi). The ultimate material strength is 61.5 (ksi). Determine its equivalent stress amplitude of a fully reversed cyclic stress.

**Solution:**

Per Equation (2.21), the equivalent stress amplitude in this example with a positive mean stress $\sigma_m = 5$ (ksi) will be:

$$\sigma_{a-eq} = \sigma_a \left( \frac{S_u}{S_u - \sigma_m} \right) = 17 \left( \frac{61.5}{61.5 - 5} \right) = 18.5 \text{ (ksi)}.$$  

**Example 2.4**

A fatigue test specimen is under cyclic stress with a mean stress $\sigma_m = 15$ (ksi) and a stress amplitude $\sigma_a = 15$ (ksi). The fatigue life, that is, the number of cycles at failure under such cyclic loading for this fatigue specimen is $6.5 \times 10^5$ (cycles). The ultimate strength of the material of the specimen is 61.5 (ksi). Express this fatigue test data as a fatigue test data under a fully reversed cyclic stress.

**Solution:**

Per Equation (2.21), the equivalent fatigue strength in this example with a positive mean stress $\sigma_m = 21$ (ksi) will be:

$$S'_f = \sigma_{a-eq} = \sigma_a \left( \frac{S_u}{S_u - \sigma_m} \right) = 21 \left( \frac{61.5}{61.5 - 21} \right) = 31.89 \text{ (ksi)}.$$  

So, for this fatigue test, the fatigue test results could be equivalently expressed by:
2. RELIABILITY OF A COMPONENT UNDER CYCLIC LOAD

- the fatigue strength $S'_f = 31.89$ ksi at the fatigue life $N = 6.5 \times 10^5$ (cycles) under a fully reversed cyclic stress, that is, $(S'_f = 31.89$ ksi, $N = 6.5 \times 10^5$ cycles).

2.6 THE FATIGUE STRESS CONCENTRATION FACTOR

The fatigue stress–concentration factor $K_f$ will be used to multiply the nominal stress amplitude and can be treated as a normally distributed random variable. Its mean $\mu_{K_f}$ can be calculated by the following equation [7]:

$$\mu_{K_f} = \frac{K_t}{1 + \frac{2}{\sqrt{r}} \left( \frac{K_t - 1}{K_t} \right) \sqrt{\alpha}}.$$  \hspace{1cm} (2.22)

where $K_t$ is static stress concentration factor which can be obtained through any design handbook and some websites. $r$ is the notch radius in the unit of inch. $\sqrt{\alpha}$ is defined as the Neuber constant and can be calculated through the following equation:

$$\sqrt{\alpha} = \begin{cases} 
\frac{5}{S_{ut}} & \text{For a transverse hole} \\
\frac{4}{S_{ut}} & \text{For a shoulder} \\
\frac{5}{S_{ut}} & \text{For a groove},
\end{cases}$$  \hspace{1cm} (2.23)

where $S_{ut}$ is material tensile ultimate strength in the unit of ksi.

The coefficient of variance of the fatigue stress concentration factor $K_f$ can be estimated by the following equation:

$$\gamma_{K_f} = \begin{cases} 
0.11 & \text{For a transverse hole} \\
0.08 & \text{For a shoulder} \\
0.13 & \text{For a groove}.
\end{cases}$$  \hspace{1cm} (2.24)

The standard deviation of the fatigue stress concentration factor $K_f$ will be:

$$\sigma_{K_f} = \mu_{K_f} \times \gamma_{K_f}.$$  \hspace{1cm} (2.25)