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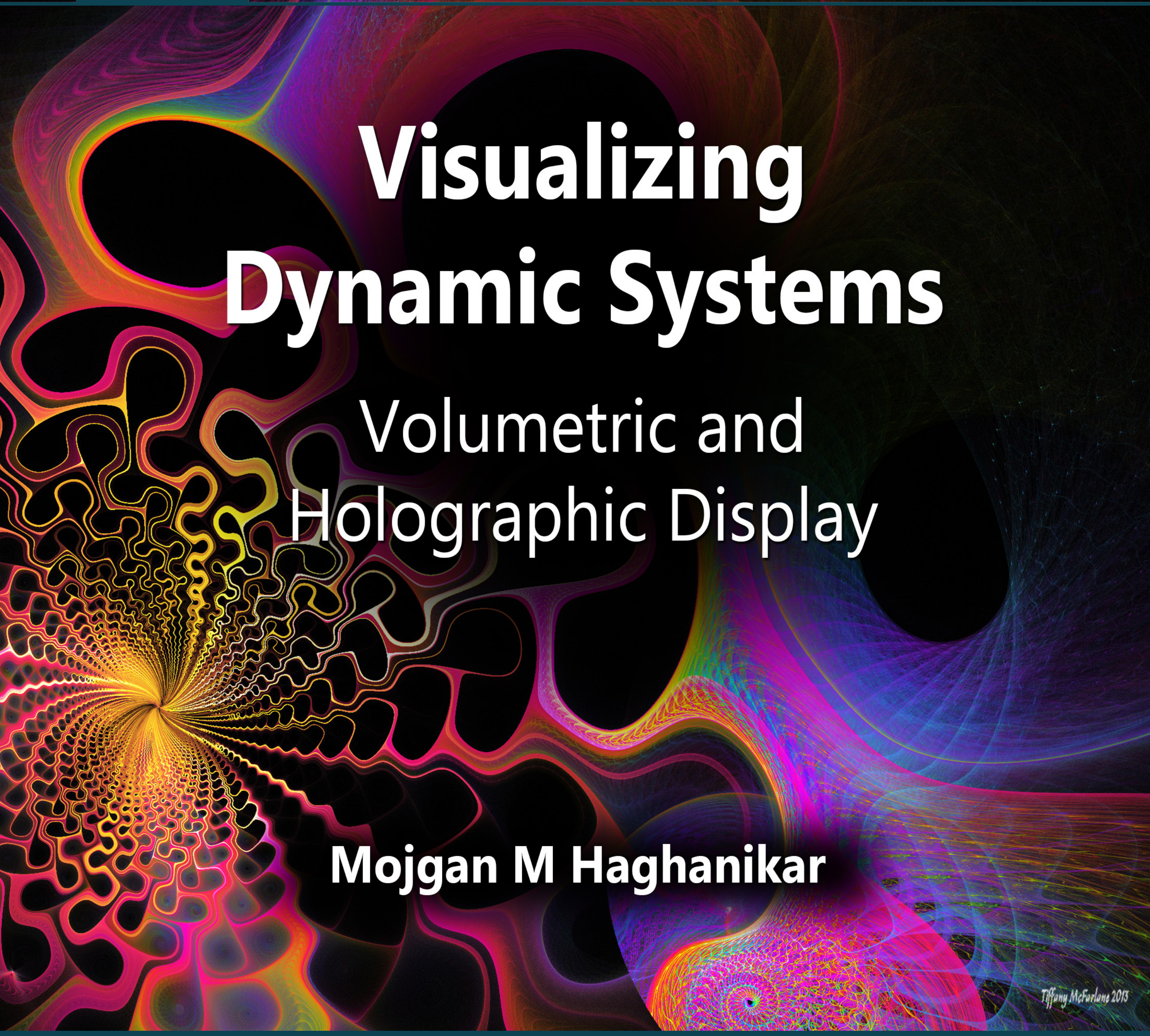

Visualizing Dynamic Systems

Volumetric and Holographic Display

Mojgan M Haghanikar

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Visualizing Dynamic Systems:

Volumetric and Holographic Display

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Visualizing Dynamic Systems:

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Mojgan M. Haghanikar
SETI Institute

SYNTHESIS LECTURES ON ENGINEERING, SCIENCE, AND TECHNOLOGY #15

MORGAN & CLAYPOOL PUBLISHERS

ABSTRACT

This book is aimed to help instructional designers, science game designers, science faculty, lab designers, and content developers in designing interactive learning experiences using emerging technologies and cyberlearning. The proposed solutions are for undergraduate and graduate scientific communication, engineering courses, scientific research communication, and workforce training.

Reviewing across the science education literature reveals various aspects of unresolved challenges or inabilities in the visualization of scientific concepts. Visuospatial thinking is the fundamental part of learning sciences; however, promoting spatial thinking has not been emphasized enough in the educational system (Hegarty, 2014)[1]. Cognitive scientists distinguish between the multiple aspects of spatial ability and stresse that various problems or disciplines require different types of spatial skills. For example, the spatial ability to visualize anatomy cross-sections is significantly associated with mental rotation skills. The same is true for physical problems that often deal with spatial representations. However, most of the physics problems are marked by dynamicity, and visualizing dynamicity is inferred by the integrations of different participating components in the system. Therefore, what is needed for learning dynamicity is visualizing the mental animation of static episodes.

This book is a leap into designing framework for using mixed reality (XR) technologies and cyberlearning in communicating advanced scientific concepts. The intention is to flesh out the cognitive infrastructure and visuospatial demands of complex systems and compare them in various contexts and disciplines. The practical implementation of emerging technology can be achieved by foreseeing each XR technology's affordances and mapping those out to the cognitive infrastructure and visuospatial demands of the content under development.

KEYWORDS

complex systems, visualization, mixed reality, spatial ability, science education, cyberlearning

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[1] M. Hegarty, "Spatial thinking in undergraduate science education," *Spatial Cognition and Computation*, 14, 142–167, 2014

I dedicate this book to all teachers and instructors around the world for their steadfast diligence, especially during the pandemic!

Contents

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Introduction

This book is a leap into designing techniques that uncovers the invisible paradigm of scientific interactions. For the first time, the frontiers of technology such as mixed realities have provided means for communicating dynamicity of scientific concepts as of the complex systems. The aim is to provide guiding principles blended from educational research, Human–Computer Interactions (HCI) and EdTech solutions to stay ahead of design practices. The targeted audiences are educators, instructional designers, science educators, science teachers, science lab instructors, cognitive scientists, UX designers, professionals across industries, workforce trainers, science and engineering instructors and researchers, content developers, andgame designers. If you are not a designer but a decision maker on an educational platform, this book provides a diligent guide for how to communicate with your designers and consultants in your team.

Before discussing the implications of emerging technologies such as Virtual Reality (VR), and Augmented Reality (AR) in scientific communication, we need a framework to start. This framework should classify various subjects' cognitive demands and find an appropriate technology to create an EdTech-assisted rendering. To this end, the first few chapters deal with a review of complex systems, the distinction of various dynamic processes and system thinking necessities, essential skills such as visuospatial skills, and a review of science education literature on challenges in visualization. The aim is to search for a classification scheme to categorize various complex systems in terms of their system thinking and spatial demand.

Design with mixed realities is cost-effective and to make the best of the design requires understanding rules in this new frontier of extended reality. A crucial part of achieving a good design is clearly understanding the objectives and users' needs and capacities. Like a filmmaker whose big shoot is coming up, you may be excited about every scene you want to create. Like making film projects that storyboard sketches the scenes, storyboarding for mixed realties is visualizing every scene of your rendering design. However, storyboarding for scientific communication is not about sharing experiences and knowledge but about the fine skills of the audience. A good storyboard needs a specific framework tailored to the audience, and that is not possible unless taking into account the previous research and evaluation that should be served as evidence-based strategies. Ultimately, a successful design is defined by an optimum balance between all the parameters involved. This book is a roadmap to bridge the previous research and experiences to a promising design with mixed reality technologies.

Overview of Chapters

CHAPTER 1: THE ART OF THINKING ABOUT COMPLEX SYSTEMS

Telescopes are a tool for the naked eye to uncover the farthest things in the sky, and microscopes reveal earthly items too small to see. Nevertheless, what are the tools needed for naked perception to uncover the behavior of nature and the invisible interactions? The ultimate goal of learning is to understand better the world and nature, which are systems with different complexity levels. The complexity is a challenge, yet our thinking is often biased and simplistic. Scientific models are tools and mental representations of natural phenomena. Comprehending the complex and nonlinear world requires a system thinking that is up to the mark with the complex system's cognitive demand. Accordingly, every new type of system is like starting a new paradigm that requires suitable and unique toolkits. In this view, various styles of thinking about systems are classified, including static, dynamic, deterministic dynamic, and chaotic. This classification scheme can be used to categorize various types of complex systems in terms of their system thinking and spatial demand.

CHAPTER 2: SPATIAL ABILITIES AND SUCCESS IN SCIENCES

Deeping dive into different aspects of visualization in sciences, this chapter provides an overview of psychometric cognitive science literature to distinguish the types of spatial abilities and highlight the visuospatial requirements central to visualization to various disciplines. We will provide an overview of cognitive studies to understand better students' challenges in visualizing spatial representations and subsequently suggest resolutions to nurture spatial thinking in sciences using emerging interactive technologies.

CHAPTER 3: SCIENCE EDUCATION LITERATURE: INVISIBLE CONCEPTS

Visualizing abstract concepts and processes is one of the most critical challenges of learning physics. This chapter aims to present an overview of visualization challenges that have been reported in physics education. The discussion is on many types of invisible concepts in sciences, comparison of mental visualization of concepts over the different time and scale, and mental visualization of dynamic processes.

CHAPTER 4: EDTECH SOLUTIONS

EdTech is a practice of implementing evidence-based teaching principles by using technology. The evidence-based practices rest on four pillars of cognitivism, constructivism, social constructivism, and visualization. Education designers use a technology genre to promote student-centered learning experiences. Student-centered practices allow users to delve into real-world scenarios and test their problem-solving skills, teamwork, communication, and collaborating abilities.

However, choosing a helpful EdTech might be like searching for a needle in a haystack. From numerous EdTech productions how can a user select a credible intervention? How may the user know if the voice of the educator was incorporated in the design? Users should have access to a credible source that provides vetted resources for a diverse set of users.

comPADRE[2] is a network of accessible online resource collections supporting faculty, students, and teachers in physics and astronomy education. The other collections of resources are PhysPort[3], which inventorized the resources developed by physics education researchers and collected the use of numerical modeling at all levels of physics education, and PICUP, a community for those promoting computation in the physics curriculum (Mason, 2012).

This chapter revisits the decades of using EdTech solutions in science. The future designs using emerging technologies can be built on previous trends. Such trends provide insight into educational designers' minds and help them fix the challenges previously faced by both students and educators.

CHAPTER 5: EMERGING TECHNOLOGIES: A TWIST IN EDTECH SOLUTIONS

The recent advancements in mixed reality developments hold significant potential to unlock visualization in three dimensions, eliminating the barriers of experimentation and interaction in e-learning. In addition to 3D allowance, mixed reality technologies enable interactions with digital objects merged into the physical world. This chapter looks at the attributes of these technologies and some current examples of how they are being used and comparing their hardware, software, and affordances that suit various purposes. The multiple facets of VR are discussed and differentiated from AR that blends the digital renderings projected over physical surroundings within people's field of vision. The discussion will follow Hololens or zSpace interactions in which an expanded field of view extends out of the screen to the surroundings and allows more possibilities for comparing and exploring the cause-and-effect relationships.

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[2] https://www.compadre.org.

[3] https://www.physport.org.

CHAPTER 6: CURRICULUM DESIGN AND EMERGING TECHNOLOGIES

The emergence of Massive Online Open Courses (MOOCS) was a game-changer in education which started with online streaming by professors at Stanford University. However, there are many aspects of constructivism that cannot be implemented employing remote courses' current means. In this view, Chapter 6 introduces a new generation of virtual science labs to present the particular affordances of cyberlearning with different resolutions to sketch a unique platform for communicating science and engineering concepts. In doing so, this chapter offers insights on selecting the type of emerging technology according to a lesson design's purpose and objectives.

CHAPTER 7: BREAKTHROUGHS IN SCIENTIFIC COMMUNICATION

Like every history book that rests upon fierce rivalry that shaped the revolution's course, this chapter discusses how the previous research of learning challenges informed our new designs with the technology genre. Discussing the challenges facing scientific communication and learning from previous developments are laying a launchpad with varying preparedness levels for fully understanding new potentials and breakthroughs. The deterioration of traditional instruction methods has set the stage for implementing a complicated set of integrated designs.

The critical attributes of the AR/VR technologies have made a breakthrough in remote science labs that could replace e-learning with cyberlearning, provide virtual benches for workforce training, and a new platform that reshapes the research and assessment.

CHAPTER 1

The Art of Thinking About Complex Systems

1.1 INTRODUCTION

The quest of science is to explore how nature works. Many aspects of nature are observables, but many other aspects are invisibles, and the only way to detect the invisible is to trace their interactions with the observables through a chain of cause and effect. The invisible aspects that escape our senses are often too far, small, fast, or slow. The physical entities are not always substances; they are also in the forms of waves and energies. Maybe we can see the standing waves in a string, but in quantum mechanics, the quantum state of a particle, which is called wave function, is invisible and defined by the probability of locating the particle at that point. We have a limited range of sensibility to detect the spectrum of waves. The other type of invisible is a pile of data. The most invisible aspect of nature is processing too much information. Through the ages, humans have invented many instruments to compensate for their limitations to observe and explore. Telescopes are designed to observe the far, microscopes for the invisible tiny structures, and computers for processing and recording bulky data. With the aid of proper instrumentations, invisible aspects of nature can be discovered, recorded, and represented. This book aims to discuss a particular type of invisibles that are not in the form of substances but in the form of patterns, interactions, interrelationships, and behaviors. The main goal is to introduce suitable instrumentations for capturing the most hidden part of nature and its complex system: the *interrelationships*, *interactions*, *links between cause and effect*, *patterns*, and *processes*.

Scientists often face a variety of perplexing challenges dealing with complex systems. The study of complex phenomena requires competency, efficient communication and collaboration across the disciplines, and interdisciplinary training. The study of natural phenomenon necessitates an interdisciplinary approach. In order to bridge the gap between cross-border research and elucidate barriers of communications, interdisciplinary teaching is often encouraged in undergraduate science (Nae, 2017; Buchbinder et al., 2005)—meaningful learning associates with the reconstruction of concepts in a new context or discipline (Bransford et al., 2000). Understanding the inherent complexity of natural phenomena draws on various disciplines and, consequently, on various representation tools and integrative research approaches. Traditional practices tend to compartmentalize the disciplines, and subsequently, the experts in each field are equipped with compartmentalized knowledge, and compartmentalized spatial and cognitive skills pertain to constructing that type of knowledge. As a general rule, is needed to distinct cognitive infrastructure hierarchies and visuospatial demands of complex systems and compare the demands that existed for various types of complex systems and inter-disciplines.

There are many examples of complex systems in nature, society, environment, climate, or living organisms. The human body, our learning activities, cell of a living organism, colony of ants and beehives, flock of birds, transmission and spread of infectious diseases, climate change, and social behaviors are examples of complex systems. Frequently, the constituents and building blocks of complex systems are also complex organisms or systems. Many processes in nature, social context, or artificially engineered can be categorized as complex systems. One way of describing the complex system is to model the system's emergent collective behavior as a whole and discern the collective performance from the function of system entities.

Complex systems in general are marked with patterns of commonalities such as nonlinearity and unpredictability. Systems can be studied in terms of their composite elements' interrelationships, and when systems are not complex, the emergent behavior can be predicted based on those interrelationships. As a result, a simple system's behavior can be modeled by the participating entities and their interrelationships, whereas in a complex system, a more significant number of interacting entities contribute to the emergent behavior, and even different systems analyzing tool or data collecting methods apply to complex systems. The superimposed interrelations of the participating entities create a convoluted outcome and not always linear and predictable as it can be affected significantly by the small random behavior of entities and the initial conditions.

Science is studying natural phenomena and understanding their functionality by seeking models that capture natural phenomena' complexity and dynamicity. To describe a system, scientific models often leverage mathematical and computational tools to simulate the system's mechanism into a new representation based on mathematical and computational algorithms. Scientists often use mathematical models to describe the data attained through observations and experimentations to perceive the dynamics and architecture. The models are mathematical tools or graphical representational tools that display interrelationships in a compact illustration with distributed information to ease mental visualization.

Numerous scientific theories have been employed to describe natural and artificial complex systems in various disciplines. However, there have been commonalities in research methodologies, modeling, and mathematical formulation. Wolfram (2020) sketched an outline for the study of complex systems to derive mathematical models that capture the essential features of the collected complex behavior generated due to many components interacting dynamically. Wolfram noted that many systems' essential components are simple but capturing many constituents' interaction transcends contemporary mathematics limits.

1.2 DYNAMIC SYSTEMS: UNVEILING THE INVISIBLE PATTERNS

Understanding a phenomenon involves constructing an internal cognitive structure that is a mental representation of an object or process, and there is a large number of possible cognitive structures that can represent a new stimulus (Sedikidies and Skoronski, 1991). As the scientific knowledge evolved through the history of science, more advanced and abstract cognitive representations emerged subsequently to model the new stimulus. Studying a dynamic system involves dynamic thinking, which requires an exemplary infrastructure of cognitive and visuospatial skills. Dynamic thinking incorporates the elements of time (Bratianu, 2007) and should be distinguished from inertial thinking that is the habit of novices and associates with static objects. Dynamic systems involve processes in motion. However, not all dynamic systems can be deterministic.

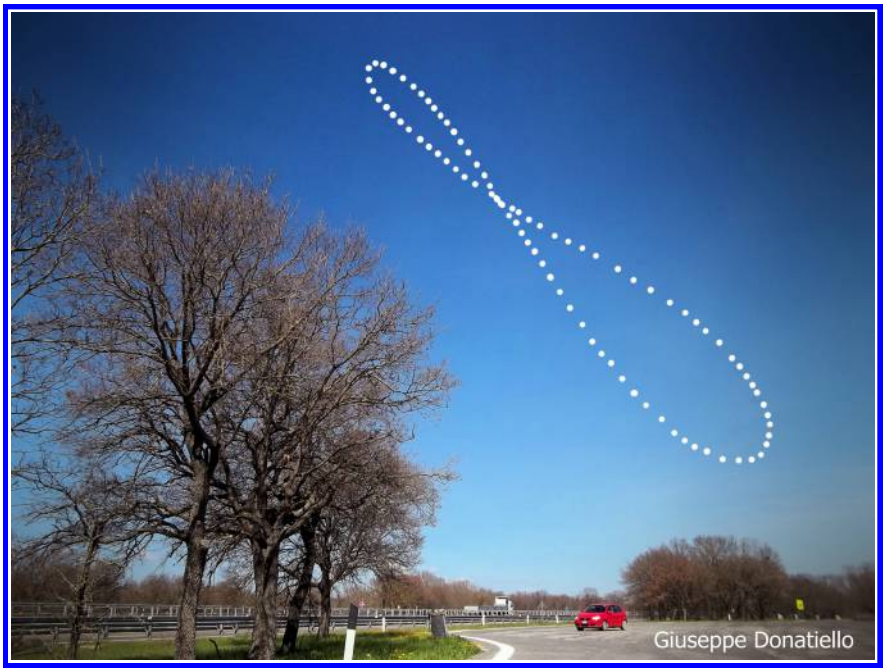

Figure 1.1: Analemma Image of Analemma by Giuseppe Donatiello is licensed under Creative Commons CC0 1.0. https://commons.wikimedia.org/wiki/File:Analemma_A14_2016_(25907420783).jpg.

For example, analemma or the "Sun's figure eight" is a diagram showing the position of the Sun in the sky as seen from a fixed location on Earth at the same mean solar time, as that position varies over a year. If the Earth's orbit around the Sun was a complete circle, the Sun would rise and set on the same path across the sky, always with the same altitude. In other words, if the Earth's axis wasn't tilted from the plane of its orbit around the Sun, the Sun's altitude would not form an analemma over a year. However, the Earth's orbit is elliptical, and the Earth's axis is tilted by roughly 23.5° to the orbit. The combination effect of earth's orbit and tilted axis makes the analemma.

Tracing the Sun's altitude that shapes an analemma represents a deterministic process, as replication yields the same pattern. In real time, all an observer can see is the Sun's altitude going up and down, but the only time the hidden cyclic pattern appears is after recording the Sun's location over a year from the exact location same time. In this way, a model is generated representing the cyclic variations of the altitude of the Sun. Visualizing the system of the Earth–Sun orbit required a cognitive representation of a 3D dynamic geometrical model and the Sun's altitude over the span of year can be defined by a differential equation.

In contrast, one can find many dynamic processes which are not deterministic. A straightforward example is the pattern of clouds that is constantly changing. I used to live in NYC and watching the sky's different patterns at sunset was always a surprise.

Figure 1.2: Sunset NYC. Courtesy: author.

For example, replicating the recording of a video time-lapse of the motion of clouds from a fixed location such as a window will differ each time as the form of clouds, and their motions and the shadows they cast on the buildings, roofs, or ground can be chaotic and unpredictable. Although the details about the motion of the clouds are chaotic, some features remain unchanged, and this is how we can distinguish a cloud.

Figure 1.3: Time-lapse video of the clouds. Reproduced with permission from: Maciej Czarnecki. Personal communication.

1.3 ARTIFICIAL INTELLIGENCE AND 3D TIME-LAPSE REPRESENTATIONS

The real essence of active learning is promoting scientific reasoning to describe a phenomenon; therefore, data acquisition and data interpretation are the core subset of inquiry. According to Symons and Boschetti (2013), the underlying patterns of dynamic systems remained hidden when an extensive tabulation of data obscure them. Instead, graphical representation of data such as charts and diagrams, the trends, patterns, and interrelationships are toolkits to unmask the hidden patterns. Infographics compacts the data and highlights the trends and facilitate the communication of research outcomes.

Machine hallucinations are the latest in watching 3D time-lapse of the large number of images taken over the span of time. Google satellite, biological and astronomical imaging advancements, and many other archiving improvements have provided access to large image databases. A great innovative artist, Anadol (2020), blended art and AI and machine learning to create a machine hallucination that is a dynamic time-lapse of 100 million photographs into 3D movies projected in large halls and theatres. Machine Hallucination was an exploration of time and space experienced through New York City's public photographic archives.

When Refik Anadol heard about the Alzheimer's disease of his beloved uncle, his coping mechanism was to remember memories in a new light. According to his TED talk (Anadol, 2020), he was determined to bring memories into life, so they were not disappearing, but they were reshaping. Therefore, he invented an immersive data visualization technique that can be applied to any big dataset distributed in the span of time and space.

Figure 1.4: Machine hallucination. Reproduced with permission from: Refik Anadol Studio.

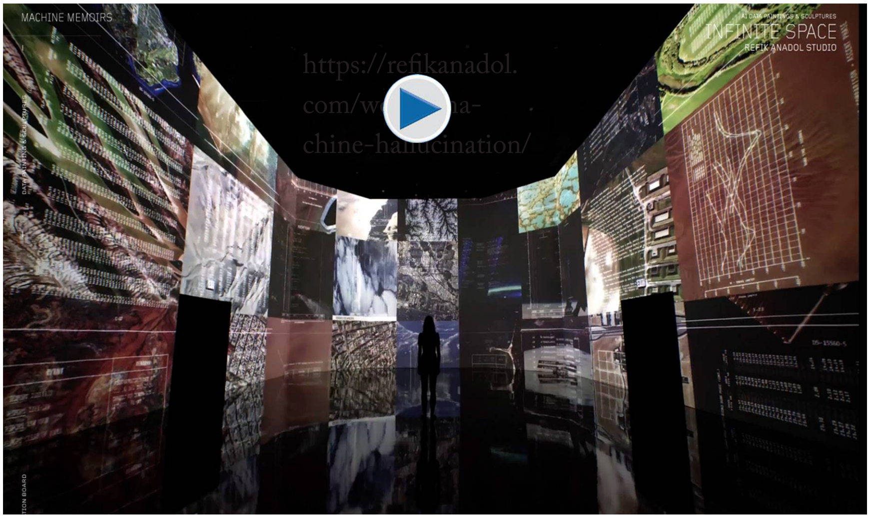

Figure 1.5: Machine Hallucination Exhibition. Reproduced with permission from: Refik Anadol Studio, https://refikanadol.com/works/machine-hallucination/.

The upshot of all these is the advancement of artificial intelligence (AI) and archiving, which have offered flexibility in tracking dynamicity in extended periods. Therefore, lengthy observations of the chaotic systems may shed new light on the chaotic behavior. Comparing the above examples, it is evident that chaotic dynamic systems do not have a deterministic pattern as it might be found for analemma but may show a pattern that can be found similar to the Mandelbrot set, which is coming next.

1.3.1 MANDELBROT SET

Benoit Mandelbrot (1982) saw a pattern for deterministic chaotic systems. In the same fashion of tracking the sun's location, he tracked the iterations of quadratic functions. He considered himself a fractal expert and was inspired by self-similarities in nature. His interest brought him to the Julia sets, which consist of values such that an arbitrarily small perturbation can cause drastic changes in the sequence of iterated function values.

The simplest definition of Julia set has been provided by McGoodwin (2000), who presented Julia sets as the family of sets generated by the special quadratic case form $f(z) = z^2 + c$. Here z represents a complex number that can take on all values in the complex plane. The constant "c" is also defined as a complex number, but it is held constant for any given Julia set. Therefore, there is an infinite number of Julia sets, each defined for a given value of c.

By iterating f(z) values as the number of iterations increases toward infinity, either f(z) can continue to grow and blow up or stay bounded. There are two group of points in the complex plane. Those that do not stay bounded with successive iterations called escape set and other points in the complex plane that stays bounded called prisoner set as many iterations are taken to infinity. All points must either be in one or the other set. The common boundary between the escape set and the prisoner set is called the Julia set.

Mandelbrot's addition to Julia's sets was to visualize the patterns of chaotic systems with the technology he had access to in 1978. As he worked for IBM, he came up with the idea to use computers to obtain the graphical representations of chaotic behavior. He graphed the iterations of quadratic functions.

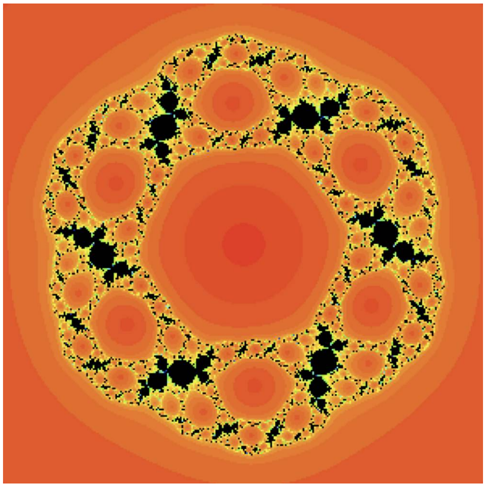

Figure 1.6: The connected Julia set for the map $z^3 + (.105 + .05i)/z^2$. Reproduced with permission from: Dr. Robert Devaney.

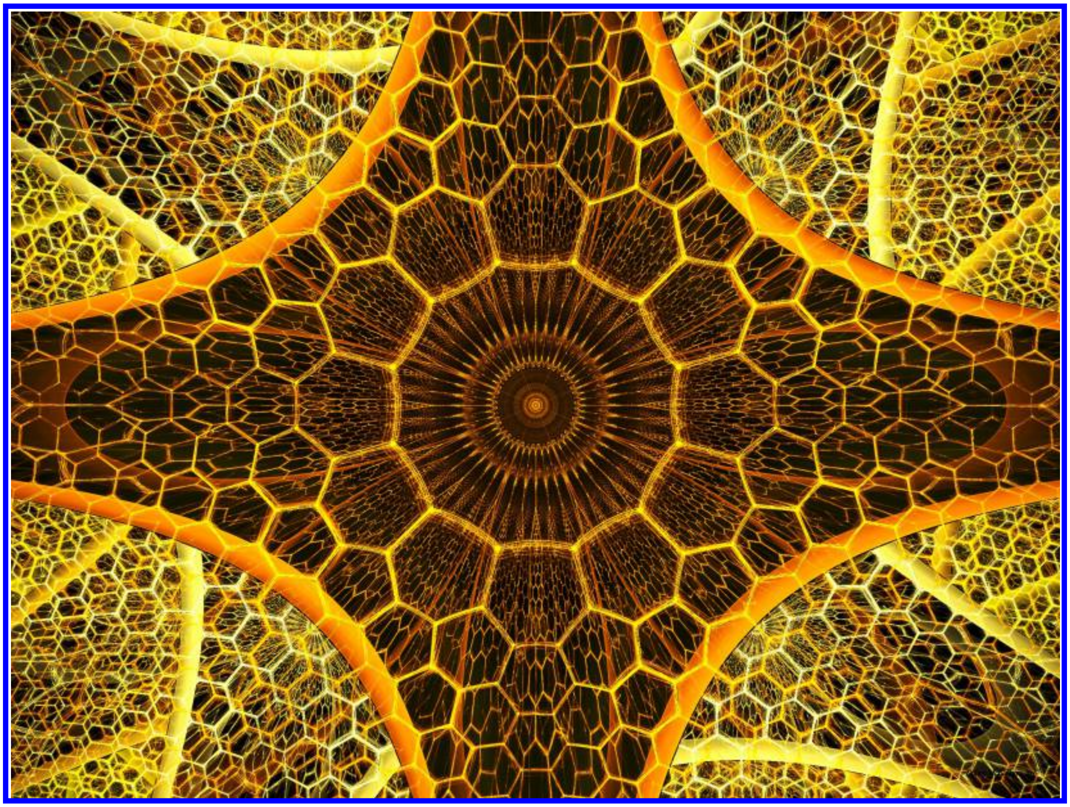

Figure 1.7: Fractal Set: Inside the hive. Reproduced with permission from: Tiffany Mcfarlane, https://www.trmdesignco.com/inside-the-hive.html.

Bear in mind the invisibility that was unmasked with Mandelbrot's innovative strategy. His work's significance was not just demonstrating the hidden patterns embedded within the chaotic iterations of quaratic equations, but a method that can be applied to any factes of chaotic systems in nature. Therefore, within the apparent randomness of complexity and chaos there are invisible patterns, interconnectedness, constant feedback loops, repetition, self-similarity, fractals, and self-organization which can also be considered as deterministic behavior. Chaos theory is often associated with many variable systems. Nevertheless, due to the 20th century's prominent discoveries on chaos and dynamic systems, the simplest type of systems can also exhibit chaotic behavior (Devaney, 2018). Scientists have begun studying simple systems' chaotic behavior to build a foundation for further research in turbulent many variable systems. Using this approach, Devaney (2018) has made chaos theory accessible to the learner by illustrating the chaotic behavior in normal and straightforward quadratic functions. Quadratic equations are easy to be solved and graphed. To add dynamicity to a quadratic equation, the function is needed to be composed upon itself, which is called an iterative process. To iterate a function, we should take the output of a function and feed it as an input to repeat the operation repeatedly (Devaney, 2018). In doing so, for some types of functions, a set of input numbers grouped as Julia sets could be found, resulting in totally bizarre, random outcomes and susceptibility to initial conditions, whereas the behaviors of other groups of numbers are predictable. The set of input numbers with the bizarre outcome that was grouped as escape set and Julia set includes values that with an arbitrarily small perturbation, a drastic change can be obtained in the sequence of iterated function values. Julia sets are an example of fractals as they create a self-similar subset over and over again. However, the patterns of chaotic systems are prone to drastic change with any minor changes of initial conditions that may substantially change the overall outcome.

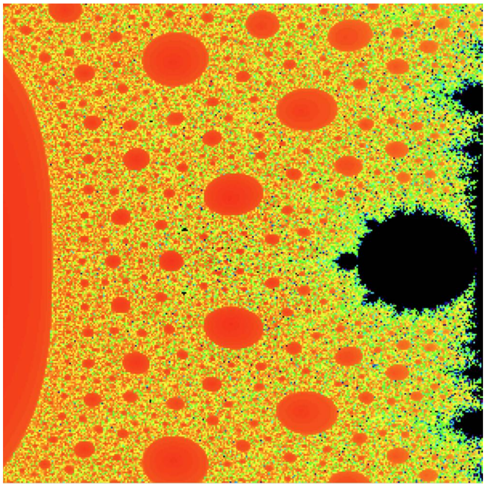

Figure 1.8: A magnification of the parameter plane for the family of maps $z^2 + C/z^2$. Reproduced with permission from: Dr. Robert Devaney.

1.4 CHAOS AND ENERGY DISSIPATION

Open systems that exchange energy with the environment can change the systems' state and initial conditions. One visually good example is a magnetic pendulum that shows the chaotic behavior under three magnets and gravitational field (Berg, 2020). The experiment was originally conducted by German scientists Hilgenfeldt and Schulz (1994). The system constitutes a magnetic pendulum sways by the net forces of gravity and magnets located on an equilateral triangle's edges. The pendulum is suspended from a pivot that is located on a vertical axis passing through the center of equilateral triangle. The pendulum's path's chaotic patterns appear after simulating the motion of a magnetic pendulum under the influence of three magnets and gravity. A color-coded pattern is obtained by tracking the pendulum assigning three colors to each magnet and recording the color of the magnet over which the pendulum came to a rest.

The pattern is dynamic, and any minor variation in the pendulum's starting point will result in a vastly different color-coded pattern. The friction cause dissipation and change of initial conditions. Like every chaotic system, the simulation results are susceptible to slight variations in their initial conditions, including the pendulum's length.

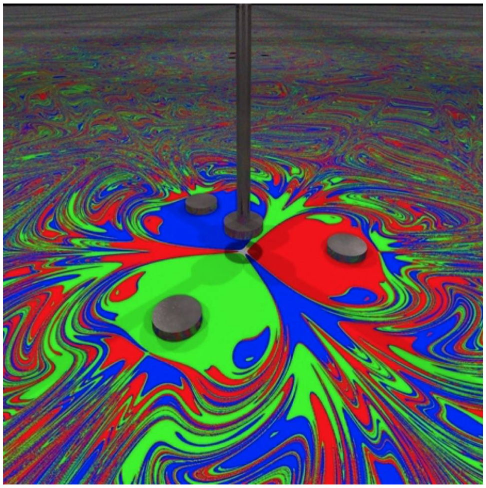

Figure 1.9: Chaotic magnetic pendulum. Reproduced with permission from: Paul Nylander (http://bugman123.com).

Figures 1.10, 1.11, 1.12, and 1.13 are several examples of various patterns obtained as a result of changing the variables such as length of pendulum or gravitation.

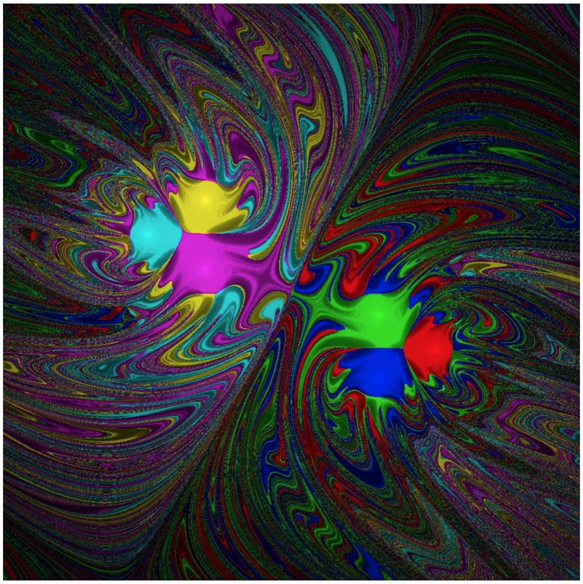

Figure 1.10: Magnetic pendulum with change of variables. Reproduced with permission from: Ingo Berg.

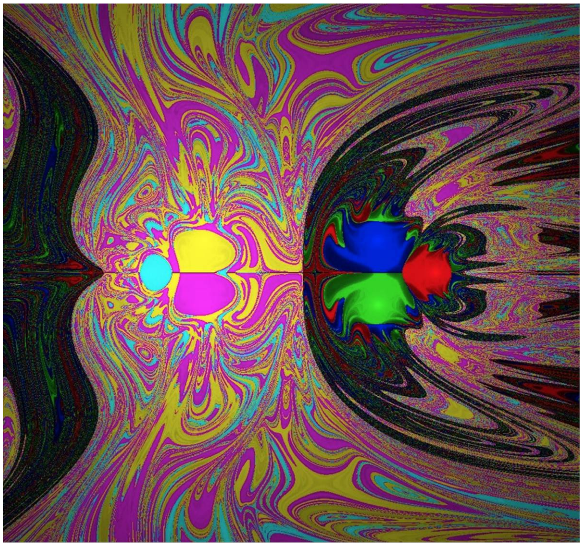

Figure 1.11: Magnetic pendulum with change of variables. Reproduced with permission from: Ingo Berg.

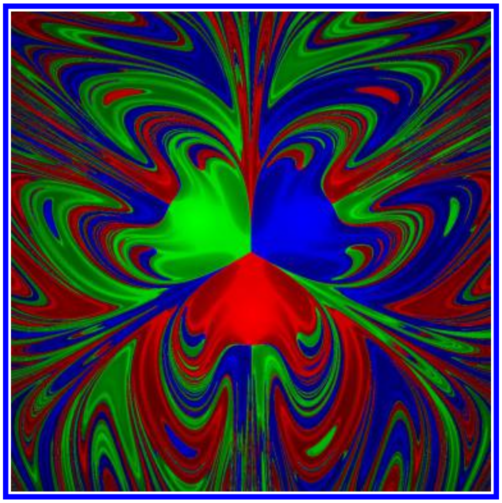

Figure 1.12: Magnetic pendulum with change of variables. Reproduced with permission from: Ingo Berg.

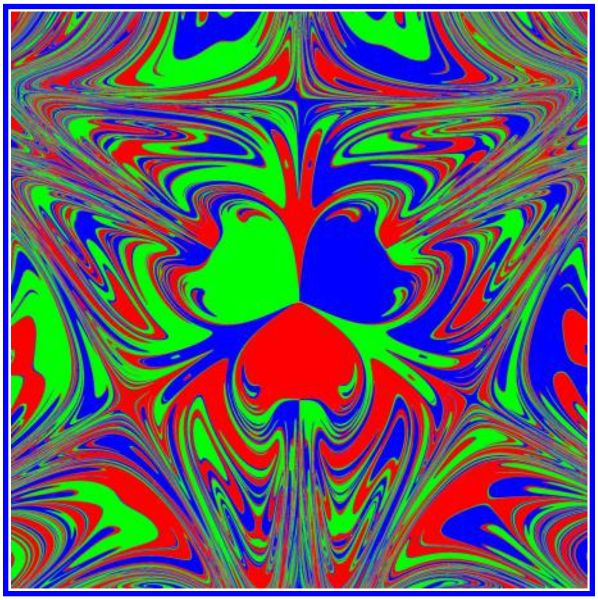

Figure 1.13: Magnetic pendulum with change of variables. Reproduced with permission from: Ingo Berg.

1.5 CHAOS IN NATURE

As discussed earlier, dynamic systems are the type of systems whose characteristics change with time, and their mathematical representations are time variant. The time-dependent functions are either differential equations or iterated functions (Feldman, 2019). Differential equations play a significant role in modeling physical sciences problems. Nevertheless, what type of behaviors represent iterated functions? Complex systems with chaotic behavior and iterated modeling also have their commonalities and distinctions. Below are examples of chaotic behavior from various domains with commonalities in mathematical modeling, including turbulent flow, crystal growth, aggregations and supramolecules, ecological and social systems. My point here is not checking out the example from a scientific view but emphasizing each's hidden aspects through cognitive lenses.

1.5.1 TURBULENT FLOW

The chaotic motion of gas flow or liquid fluids are examples of a complex system. Fluid motion can be laminar or turbulent. As one of the paragons of chaotic systems, turbulent flow can be found everywhere. There is a turbulent flow when water starts boiling in a pot until rhythmic ripples start to appear or when the draining water in the bathtub start swirling.

Figure 1.14: Imaging candle flow with Schlieren optics set up. Reproduced with permission from: https://commons.wikimedia.org/wiki/File:Laminar-turbulent_transition.jpg.

Either of two types of flow may occur depending on the fluid's velocity, density, and viscosity: laminar flow or turbulent flow. Looking at the rising smoke from a cigarette illustrates the difference between the chaotic and laminar flow. Laminar is an orderly flow that is associated with lower velocities, and turbulent flow is a less orderly flow. The flow patterns can be predicted by Reynold's number which is a dimensionless number determined by the ratio of inertial forces and resistant forces to deformation.

A Schlieren optics set up can be used to make visible the warm convection currents rising from a candle flame or cold air sinking from a glass of ice water. A turbulent flow motion is an unsteady flow and its flow properties throughout the location of the flow change with time.

Atcheson (2007) suggested an exciting method to reconstruct the 3D flow by capturing multiple viewpoints with Schlieren mirrors then blending the images topographically to reconstruct 3D models of the fluid's temperature distribution. To model the flow, particle image velocimetry (PIC) can be used. In this method, tiny colorful seeds can be traced, which are injected into the flow.

Put differently, turbulent flow represents chaotic behavior in terms of changes of its characteristics such as pressure, flow velocity, and concentration. For example, the mathematical model of blood flow on blood vessels may depend on a vessel's size, elasticity, diffusion, and osmosis properties or the pulsations' frequency. For a hydraulic damper, modeling depends on many subsets such as the hydraulic chamber's length and valves' pressure.

1.5.2 DENDRITE GROWTH

Dendrite growth is one of the very familiar manifestation of a chaotic behavior and fractals that resembles the beautiful patterns of tree leaves, snowflake structure, or earth crust minerals and dendrite crystalline structures are examples of dendrite growth. Dendrite crystallization forms a natural fractal pattern that can be modeled by fractal mathematics. Sethian and Strain (1992) derived a numerical model for dendrite crystal growth. They started with metastable liquid under the freezing point and added a tiny seed of the solid phase of the material under study into the liquid. After the solid phase grows, it initiates a rapid formation of dendrites, stretching branches to colder regions of liquid. To model a crystal growth, one should consider parameters like crystalline anisotropy, the liquid's viscosity, time-dependent boundary conditions, time-dependent temperature, time-dependent viscosity of flow, surface tensions, and driving forces, and the kinematics of the particles involved.

Modeling dendrites through computation have been demystified the science behind different manifestations of dendrite in nature such as tree leaves and snowflakes and it was instrumental to the neuroscience research.

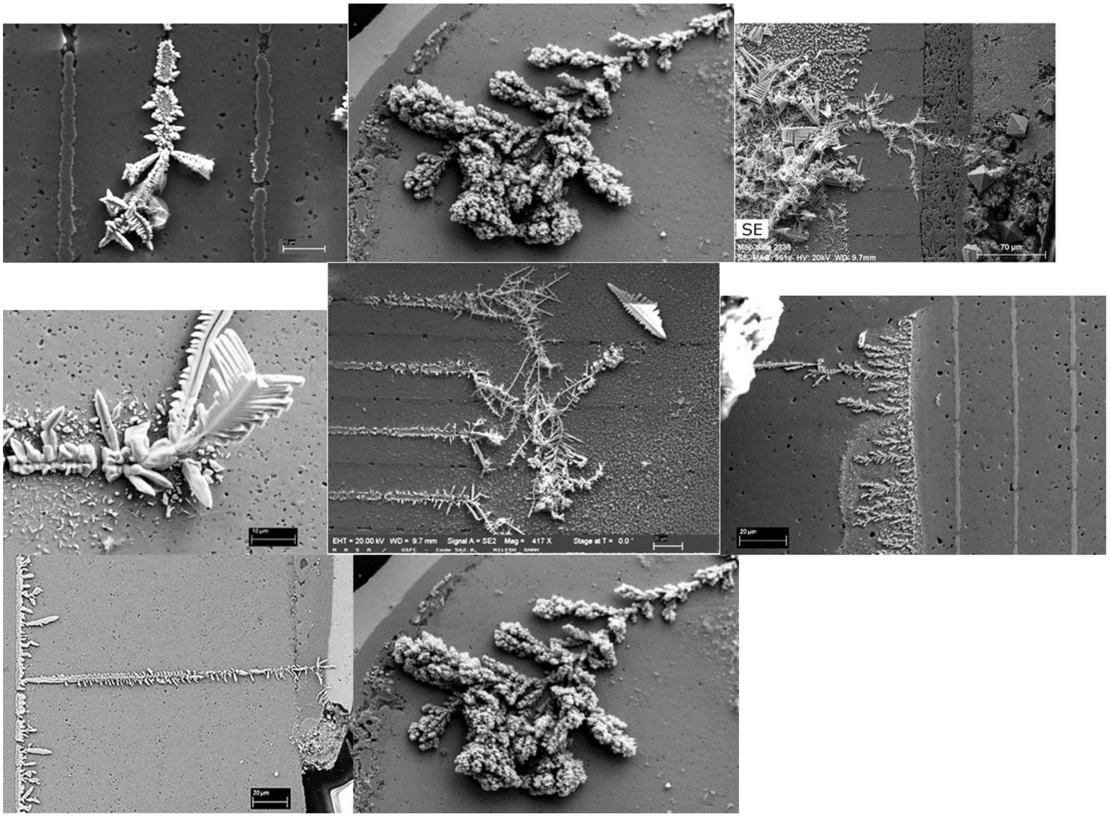

Figure 1.15: Dendrite growth in ceramic capacitors. Reproduced with permission from: Alexander Teverovsky NASA Space Technology: https://nepp.nasa.gov/files/24618/Dendrite%20growth%20in%20BME%20and%20PME%20ceramic%20capacitors%20CARTS2013_n195.pdf.